Exercises for Section 6.1

The symbol [graphing technology icon] indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system.

Click on [S] to view the complete solution of the exercise.

Click on [M] to print an enlarged copy of the graph.

In Exercises 1–8, verify the solution of the differential equation.

	Solution	*Differential Equation*
1.	$y = Ce^{4x}$	$y' = 4y$
2.	$y = e^{-x}$	$3y' + 4y = e^{-x}$
3.	$x^2 + y^2 = Cy$	$y' = 2xy/(x^2 - y^2)$
4.	$y^2 - 2 \ln y = x^2$	$\dfrac{dy}{dx} = \dfrac{xy}{y^2 - 1}$
5.	$y = C_1 \cos x + C_2 \sin x$	$y'' + y = 0$
6.	$y = C_1 e^{-x} \cos x + C_2 e^{-x} \sin x$	$y'' + 2y' + 2y = 0$
7.	$y = -\cos x \ln\lvert\sec x + \tan x\rvert$	$y'' + y = \tan x$
8.	$y = \frac{2}{5}(e^{-2x} + e^x)$	$y'' + 2y' = 2e^x$

In Exercises 9–12, verify the particular solution of the differential equation.

	Solution	*Differential Equation and Initial Condition*
9.	$y = \sin x \cos x - \cos^2 x$	$2y + y' = 2\sin(2x) - 1$, $y\left(\dfrac{\pi}{4}\right) = 0$
10.	$y = \frac{1}{2}x^2 - 4\cos x + 2$	$y' = x + 4\sin x$, $y(0) = -2$
11.	$y = 6e^{-2x^2}$	$y' = -4xy$, $y(0) = 6$
12.	$y = e^{-\cos x}$	$y' = y \sin x$, $y\left(\dfrac{\pi}{2}\right) = 1$

In Exercises 13–18, determine whether the function is a solution of the differential equation $y^{(4)} - 16y = 0$.

13. $y = 3\cos x$
14. $y = 3\cos 2x$
15. $y = e^{-2x}$
16. $y = 5\ln x$
17. $y = C_1e^{2x} + C_2e^{-2x} + C_3\sin 2x + C_4\cos 2x$
18. $y = 3e^{2x} - 4\sin 2x$

In Exercises 19–24, determine whether the function is a solution of the differential equation $xy' - 2y = x^3e^x$.

19. $y = x^2$
20. $y = x^2e^x$
21. $y = x^2(2 + e^x)$
22. $y = \sin x$

In Exercises 25–28, some of the curves different values of C in the general solution equation are given. Find the particular so through the point shown on the graph.

	Solution	*Differential Equation*
25.	$y = Ce^{-x/2}$	$2y' + y = 0$
26.	$y(x^2 + y) = C$	$2xy + (x^2 + 2y)y' =$
27.	$y^2 = Cx^3$	$2xy' - 3y = 0$
28.	$2x^2 - y^2 = C$	$yy' - 2x = 0$

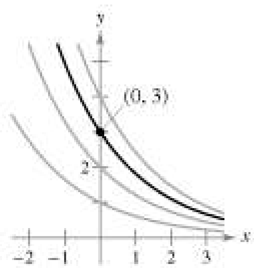


Figure for 25

Figure for 2

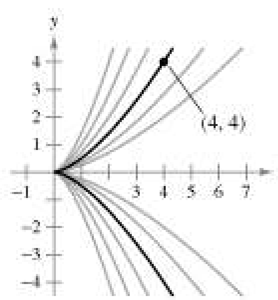


Figure for 27

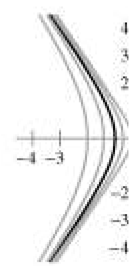


Figure for 2

In Exercises 29 and 30, the general solution equation is given. Use a graphing utility to gr solutions for the given values of C.

29. $4yy' - x = 0$
$4y^2 - x^2 = C$
$C = 0, C = \pm 1, C = \pm 4$

30. $yy' + x$
$x^2 + y^2$
$C = 0,$

In Exercises 31–36, verify that the general so differential equation. Then find the partic satisfies the initial condition.

31. $y = Ce^{-2x}$
$y' + 2y = 0$
$y = 3$ when $x = 0$

32. $3x^2 + 2$
$3x + 2y$
$y = 3$ w

33. $y = C_1 \sin 3x + C_2 \cos 3x$

34. $y = C_1$

35. $y = C_1x + C_2x^3$
$x^2y'' - 3xy' + 3y = 0$
$y = 0$ when $x = 2$
$y' = 4$ when $x = 2$

36. $y = e^{2x/3}(C_1 + C_2x)$
$9y'' - 12y' + 4y = 0$
$y = 4$ when $x = 0$
$y = 0$ when $x = 3$

In Exercises 37–48, use integration to find a general solution of the differential equation.

37. $\frac{dy}{dx} = 3x^2$

38. $\frac{dy}{dx} = x^3 - 4x$

39. $\frac{dy}{dx} = \frac{x}{1 + x^2}$

40. $\frac{dy}{dx} = \frac{e^x}{1 + e^x}$

41. $\frac{dy}{dx} = \frac{x - 2}{x}$

42. $\frac{dy}{dx} = x \cos x^2$

43. $\frac{dy}{dx} = \sin 2x$

44. $\frac{dy}{dx} = \tan^2 x$

45. $\frac{dy}{dx} = x\sqrt{x - 3}$

46. $\frac{dy}{dx} = x\sqrt{5 - x}$

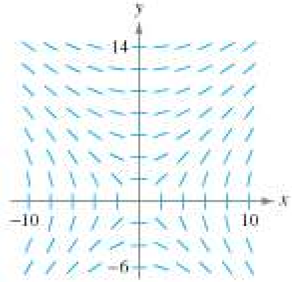
47. $\frac{dy}{dx} = xe^{x^2}$

48. $\frac{dy}{dx} = 5e^{-x/2}$

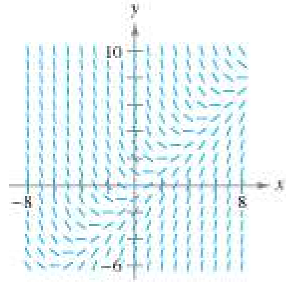
Slope Fields **In Exercises 49–52, a differential equation and its slope field are given. Determine the slopes (if possible) in the slope field at the points given in the table.**

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx						

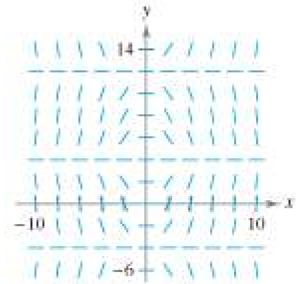
49. $\frac{dy}{dx} = \frac{x}{y}$



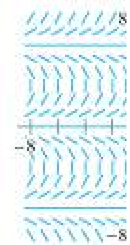
50. $\frac{dy}{dx} = x - y$



51. $\frac{dy}{dx} = x \cos \frac{\pi y}{8}$

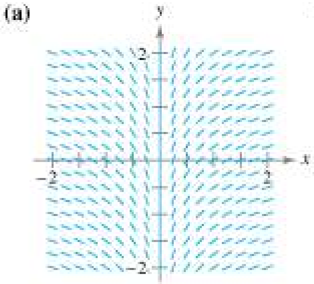


52. $\frac{dy}{dx} =$ ta



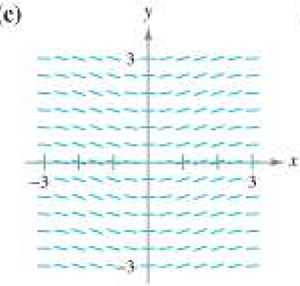
In Exercises 53–56, match the differential
slope field. [The slope fields are labeled (a), (

(a)

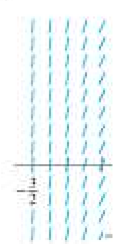


(b)

(c)



(d)



53. $\frac{dy}{dx} = \cos(2x)$

54. $\frac{dy}{dx} = \frac{1}{2}$

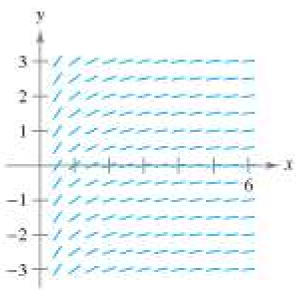
55. $\frac{dy}{dx} = e^{-2x}$

56. $\frac{dy}{dx} = \frac{1}{x}$

Slope Fields **In Exercises 57–60, (a) sketch**
the differential equation, (b) use the slope
solution that passes through the given point,
graph of the solution as $x \to \infty$ and $x \to -\infty$

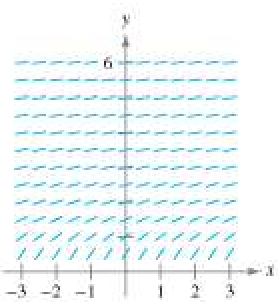
	Differential Equation	*Point*
57.	$y' = -x + 1$	$(2, 4)$
58.	$y' = \frac{1}{3}x^2 - \frac{1}{2}x$	$(1, 1)$
59.	$y' = y - 2x$	$(1, 1)$
60.	$y' = y + xy$	$(0, 4)$

M S **61.** ***Slope Field*** Use the slope field for the differential equation $y' = 1/x$, where $x > 0$, to sketch the graph of the solution that satisfies each given initial condition. Then make a conjecture about the behavior of a particular solution of $y' = 1/x$ as $x \to \infty$. To print an enlarged copy of the graph, select the MathGraph button.



(a) $(1, 0)$ (b) $(2, -1)$

M **62.** ***Slope Field*** Use the slope field for the differential equation $y' = 1/y$, where $y > 0$, to sketch the graph of the solution that satisfies each initial condition. Then make a conjecture about the behavior of a particular solution of $y' = 1/y$ as $x \to \infty$. To print an enlarged copy of the graph, select the MathGraph button.



(a) $(0, 1)$ (b) $(1, 1)$

Slope Fields **In Exercises 63–68, use a computer algebra system to (a) graph the slope field for the differential equation and (b) graph the solution satisfying the specified initial condition.**

S **63.** $\frac{dy}{dx} = 0.5y, \quad y(0) = 6$

64. $\frac{dy}{dx} = 2 - y, \quad y(0) = 4$

S **65.** $\frac{dy}{dx} = 0.02y(10 - y), \quad y(0) = 2$

66. $\frac{dy}{dx} = 0.2x(2 - y), \quad y(0) = 9$

S **67.** $\frac{dy}{dx} = 0.4y(3 - x), \quad y(0) = 1$

68. $\frac{dy}{dx} = \frac{1}{2}e^{-x/8} \sin \frac{\pi y}{4}, \quad y(0) = 2$

Euler's Method **In Exercises 69–74, use I make a table of values for the approxima differential equation with the specified initial of size *h*.**

S **69.** $y' = x + y, \quad y(0) = 2, \quad n = 10, \quad h = 0$

70. $y' = x + y, \quad y(0) = 2, \quad n = 20, \quad h = 0$

S **71.** $y' = 3x - 2y, \quad y(0) = 3, \quad n = 10, \quad h =$

72. $y' = 0.5x(3 - y), \quad y(0) = 1, \quad n = 5, \quad h$

S **73.** $y' = e^{xy}, \quad y(0) = 1, \quad n = 10, \quad h = 0.1$

74. $y' = \cos x + \sin y, \quad y(0) = 5, \quad n = 10,$

In Exercises 75–77, complete the table using of the differential equation and two approx using Euler's Method to approximate the pa the differential equation. Use $h = 0.2$ and 0.1 approximation to four decimal places.

x	0	0.2	0.4	0.6	0.8	1.(
$y(x)$ (exact)						
$y(x)$ ($h = 0.2$)						
$y(x)$ ($h = 0.1$)						

	Differential Equation	*Initial Condition*	*Exact Solutic*
S **75.**	$\frac{dy}{dx} = y$	$(0, 3)$	$y = 3\epsilon$
76.	$\frac{dy}{dx} = \frac{2x}{y}$	$(0, 2)$	$y = \sqrt{}$
S **77.**	$\frac{dy}{dx} = y + \cos(x)$	$(0, 0)$	$y = \frac{1}{2}($

78. Compare the values of the approximations with the values given by the exact solution. change as h increases?

S **79.** ***Temperature*** At time $t = 0$ minutes, the object is 140°F. The temperature of the obje rate given by the differential equation

$$\frac{dy}{dt} = -\frac{1}{2}(y - 72).$$

(a) Use a graphing utility and Euler's Met the particular solutions of this diffe $t = 1, 2,$ and 3. Use a step size of h utility program for Euler's Method i website *college.hmco.com*.)

(b) Compare your results with the exact so

$$y = 72 + 68e^{-t/2}.$$

80. ***Temperature*** Repeat Exercise 79 usi $h = 0.05$. Compare the results.

Writing About Concepts

81. In your own words, describe the difference between a general solution of a differential equation and a particular solution.

82. Explain how to interpret a slope field.

83. Describe how to use Euler's Method to approximate the particular solution of a differential equation.

84. It is known that $y = Ce^{kx}$ is a solution of the differential equation $y' = 0.07y$. Is it possible to determine C or k from the information given? If so, find its value.

True or False? **In Exercises 85–88, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

85. If $y = f(x)$ is a solution of a first-order differential equation, then $y = f(x) + C$ is also a solution.

86. The general solution of a differential equation is $y = -4.9x^2 + C_1x + C_2$. To find a particular solution, you must be given two initial conditions.

87. Slope fields represent the general solutions of differential equations.

88. A slope field shows that the slope at the point (1, 1) is 6. This slope field represents the family of solutions for the differential equation $y' = 4x + 2y$.

89. ***Error and Euler's Method*** The exact solution of the differential equation

$$\frac{dy}{dx} = -2y$$

where $y(0) = 4$, is $y = 4e^{-2x}$.

(a) Use a graphing utility to complete the table, where y is the exact value of the solution, y_1 is the approximate solution using Euler's Method with $h = 0.1$, y_2 is the approximate solution using Euler's Method with $h = 0.2$, e_1 is the absolute error $|y - y_1|$, e_2 is the absolute error $|y - y_2|$, and r is the ratio e_1/e_2.

x	0	0.2	0.4	0.6	0.8	1
y						
y_1						
y_2						
e_1						
e_2						
r						

(b) What can you conclude about the ratio r as h changes?

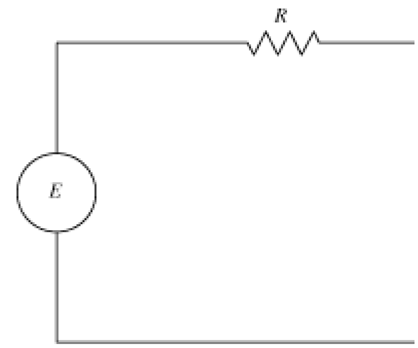
(c) Predict the absolute error when $h = 0.05$.

90. ***Error and Euler's Method*** Repeat Exe
exact solution of the differential equation

$$\frac{dy}{dx} = x - y$$

where $y(0) = 1$, is $y = x - 1 + 2e^{-x}$.

91. ***Electric Circuits*** The diagram shows a si
consisting of a power source, a resistor, an



A model of the current I, in amperes (A), a
the first-order differential equation

$$L\frac{dI}{dt} + RI = E(t)$$

where $E(t)$ is the voltage (V) produced by
is the resistance, in ohms (Ω), and L is the in
(H). Suppose the electric circuit consists
source, a 12-Ω resistor, and a 4-H inductor

(a) Sketch a slope field for the differential

(b) What is the limiting value of the curren

92. ***Think About It*** It is known that $y = e^{kt}$
differential equation $y'' - 16y = 0$. Find th

93. ***Think About It*** It is known that $y = A$ si
the differential equation $y'' + 16y = 0$. Fin

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94. Let f be a twice-differentiable real-valued

$$f(x) + f''(x) = -xg(x)f'(x)$$

where $g(x) \geq 0$ for all real x. Prove that $|f$

95. Prove that if the family of integral curve
equation

$$\frac{dy}{dx} + p(x)y = q(x), \qquad p(x) \cdot q(x) \neq 0$$

is cut by the line $x = k$, the tangents at th
tion are concurrent.