1.

Verify the solution of the differential equation.

Solution	*Differential Equation*
$y = Ce^{4x}$	$y' = 4y$

1. Check: $y' = 4Ce^{4x}$
2. $= 4y$

3.

Verify the solution of the differential equation.

Solution	*Differential Equation*
$x^2 + y^2 = Cy$	$y' = \dfrac{2xy}{(x^2 - y^2)}$

1. Check: $2x + 2yy' = Cy'$
2. $y' = \dfrac{-2x}{(2y - C)}$
3. $y' = \dfrac{-2xy}{2y^2 - Cy}$
4. $= \dfrac{-2xy}{2y^2 - (x^2 + y^2)}$
5. $= \dfrac{-2xy}{y^2 - x^2}$
6. $= \dfrac{2xy}{x^2 - y^2}$

5.

Verify the solution of the differential equation.

Solution	*Differential Equation*
$y = C_1 \cos x + C_2 \sin x$	$y'' + y = 0$

1. Check: $y' = -C_1 \sin x + C_2 \cos x$
2. $y'' = -C_1 \cos x - C_2 \sin x$
3. $y'' + y = -C_1 \cos x - C_2 \sin x + C_1 \cos x + C_2 \sin x$
4. $= 0$

7.

Verify the solution of the differential equation.

Solution	*Differential Equation*
$y = -\cos x \ln\lvert\sec x + \tan x\rvert$	$y'' + y = \tan x$

1. $y' = (-\cos x)\dfrac{1}{\sec x + \tan x}(\sec x \cdot \tan x + \sec^2 x) + \sin x \ln\lvert\sec x + \tan x\rvert$
2. $\quad = \dfrac{(-\cos x)}{\sec x + \tan x}(\sec x)(\tan x + \sec x) + \sin x \ln\lvert\sec x + \tan x\rvert$
3. $\quad = -1 + \sin x \ln\lvert\sec x + \tan x\rvert$
4. $y'' = (\sin x)\dfrac{1}{\sec x + \tan x}(\sec x \cdot \tan x + \sec^2 x) + \cos x \ln\lvert\sec x + \tan x\rvert$
5. $\quad = (\sin x)(\sec x) + \cos x \ln\lvert\sec x + \tan x\rvert$
6. Substituting,

   $y'' + y = (\sin x)(\sec x) + \cos x \ln\lvert\sec x + \tan x\rvert - \cos x \ln\lvert\sec x + \tan x\rvert$
7. $\quad = \tan x.$

9.

Verify the particular solution of the differential equation.

Solution	*Differential Equation and Initial Condition*
$y = \sin x \cos x - \cos^2 x$	$2y + y' = 2\sin(2x) - 1$
	$y\left(\frac{\pi}{4}\right) = 0$

1. $y' = -\sin^2 x + \cos^2 x + 2\cos x \sin x$
2. $\quad = -1 + 2\cos^2 x + \sin 2x$
3. Differential equation:
4. $2y + y' = 2(\sin x \cos x - \cos^2 x) + (-1 + 2\cos^2 x + \sin 2x)$
5. $\quad = 2\sin x \cos x - 1 + \sin 2x$
6. $\quad = 2\sin 2x - 1$
7. Initial condition:

$y\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{4} - \cos^2\frac{\pi}{4}$

8. $\quad = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2}\right)^2$
9. $\quad = 0$

11.

Verify the particular solution of the differential equation.

Solution	*Differential Equation and Initial Condition*
$y = 6e^{-2x^2}$	$y' = -4xy$
	$y(0) = 6$

1. $y' = 6e^{-2x^2}(-4x)$
2. $\quad = -24xe^{-2x^2}$
3. Differential equation:

   $y' = -24xe^{-2x^2}$
4. $\quad = -4x(6e^{-2x^2})$
5. $\quad = -4xy$
6. Initial condition:

   $y(0) = 6e^{-2(0)^2}$
7. $\quad = 6e^0$
8. $\quad = 6(1)$
9. $\quad = 6$

13.

Determine whether the function is a solution of the differential equation $y^{(4)} - 16y = 0$.

$y = 3\cos x$

1. $y^{(4)} = 3\cos x$
2. $y^{(4)} - 16y = -45\cos x$
3. $\quad \neq 0$
4. No

15.

Determine whether the function is a solution of the differential equation $y^{(4)} - 16y = 0$.

$y = e^{-2x}$

1. $y^{(4)} = 16e^{-2x}$
2. $y^{(4)} - 16y = 16e^{-2x} - 16e^{-2x}$
3. $= 0$
4. Yes

17.

Determine whether the function is a solution of the differential equation $y^{(4)} - 16y = 0$.

$y = C_1e^{2x} + C_2e^{-2x} + C_3 \sin 2x + C_4 \cos 2x$

1. $y^{(4)} = 16C_1e^{2x} + 16C_2e^{-2x} + 16C_3 \sin 2x + 16C_4 \cos 2x$
2. $y^{(4)} - 16y = 0$
3. Yes

19.

Determine whether the function is a solution of the differential equation $xy' - 2y = x^3e^x$.

$y = x^2$

1. $y' = 2x$
2. $xy' - 2y = x(2x) - 2(x^2)$
3. $= 0$
4. $\neq x^3e^x$
5. No

21.

Determine whether the function is a solution of the differential equation $xy' - 2y = x^3e^x$.

$y = x^2(2 + e^x)$

1. $y' = x^2(e^x) + 2x(2 + e^x)$
2. $xy' - 2y = x[x^2e^x + 2xe^x + 4x] - 2[x^2e^x + 2x^2]$
3. $= x^3e^x$
4. Yes

23.

Determine whether the function is a solution of the differential equation $xy' - 2y = x^3e^x$.

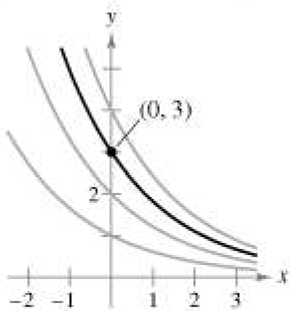
$y = \ln x$

1. $y' = \frac{1}{x}$
2. $xy' - 2y = x\left(\frac{1}{x}\right) - 2 \ln x$
3. $\neq x^3e^x$
4. No

25

Some of the curves corresponding to different values of C in the general solution of the differential equation are given. Find the particular solution that passes through the point shown on the graph.

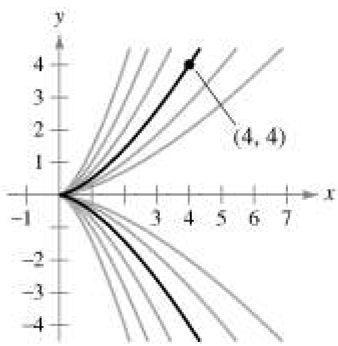
Solution	*Differential Equation*
$y = Ce^{-x/2}$	$2y' + y = 0$



1. $y = Ce^{-x/2}$ passes through (0, 3).
2. $3 = Ce^{0}$
3. $\quad = C$
4. $\quad \Rightarrow C = 3$
5. Particular solution: $y = 3e^{-x/2}$

Some of the curves corresponding to different values of C in the general solution of the differential equation are given. Find the particular solution that passes through the point shown on the graph.

Solution	*Differential Equation*
$y^2 = Cx^3$	$2xy' - 3y = 0$

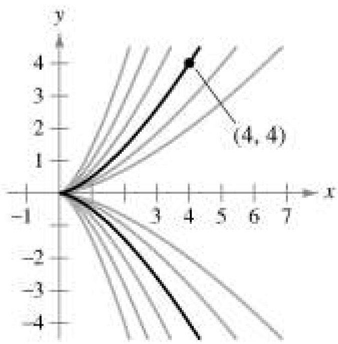


1. $y^2 = Cx^3$ passes through (4, 4).
2. $16 = C(64)$
3. $\Rightarrow C = \frac{1}{4}$
4. Particular solution: $y^2 = \frac{1}{4}x^3$
5. or $4y^2 = x^3$

27.

Some of the curves corresponding to different values of C in the general solution of the differential equation are given. Find the particular solution that passes through the point shown on the graph.

Solution	*Differential Equation*
$y^2 = Cx^3$	$2xy' - 3y = 0$



1. $y^2 = Cx^3$ passes through $(4, 4)$.
2. $16 = C(64)$
3. $\Rightarrow C = \frac{1}{4}$
4. Particular solution: $y^2 = \frac{1}{4}x^3$
5. or $4y^2 = x^3$

29.

The general solution of the differential equation is given. Use a graphing utility to graph the particular solutions for the given values of C.

$4yy' - x = 0$

$4y^2 - x^2 = C$

$C = 0,\ C = \pm 1,\ C = \pm 4$

1. Particular solutions: $C = 0$, Two intersecting lines
2. $C = \pm 1,\ C = \pm 4$, Hyperbolas

31.

Verify that the general solution satisfies the differential equation. Then find the particular solution that satisfies the initial condition.

$y = Ce^{-2x}$

$y' + 2y = 0$

$y = 3$ when $x = 0$

1. $y' + 2y = C(-2)e^{-2x} + 2(Ce^{-2x})$
2. $= 0$
3. Initial condition: $y(0) = 3$
4. $3 = Ce^{0}$
5. $= C$
6. Particular solution: $y = 3e^{-2x}$

33.

Verify that the general solution satisfies the differential equation. Then find the particular solution that satisfies the initial condition.

$y = C_1 \sin 3x + C_2 \cos 3x$

$y'' + 9y = 0$

$y = 2$ when $x = \frac{\pi}{6}$

$y' = 1$ when $x = \frac{\pi}{6}$

1. $y' = 3C_1 \cos 3x - 3C_2 \sin 3x,$
2. $y'' = -9C_1 \sin 3x - 9C_2 \cos 3x$
3. $y'' + 9y = (-9C_1 \sin 3x - 9C_2 \cos 3x) + 9(C_1 \sin 3x + C_2 \cos 3x)$
4. $= 0$
5. Initial conditions: $y\left(\frac{\pi}{6}\right) = 2,\ y'\left(\frac{\pi}{6}\right) = 1$
6. $2 = C_1 \sin\left(\frac{\pi}{2}\right) + C_2 \cos\left(\frac{\pi}{2}\right)$
7. $\Rightarrow C_1 = 2$
8. $y' = 3C_1 \cos 3x - 3C_2 \sin 3x$
9. $1 = 3C_1 \cos\left(\frac{\pi}{2}\right) - 3C_2 \sin\left(\frac{\pi}{2}\right)$
10. $= -3C_2$
11. $\Rightarrow C_2 = -\frac{1}{3}$
12. Particular solution: $y = 2 \sin 3x - \frac{1}{3} \cos 3x$

35.

Verify that the general solution satisfies the differential equation. Then find the particular solution that satisfies the initial condition.

$y = C_1 x + C_2 x^3$

$x^2 y'' - 3xy' + 3y = 0$

$y = 0$ when $x = 2$

$y' = 4$ when $x = 2$

1. $y' = C_1 + 3C_2 x^2$
2. $y'' = 6C_2 x$
3. $x^2 y'' - 3x\,y' + 3y = x^2(6C_2 x) - 3x(C_1 + 3C_2 x^2) + 3(C_1 x + C_2 x^3)$
4. $= 0$
5. Initial conditions: $y(2) = 0,\ y'(2) = 4$
6. $0 = 2C_1 + 8C_2$
7. $y' = C_1 + 3C_2 x^2$
8. $4 = C_1 + 12C_2$
9. $\left.\begin{array}{l} C_1 + 4C_2 = 0 \\ C_1 + 12C_2 = 4 \end{array}\right\}$
10. $C_2 = \frac{1}{2}$
11. $C_1 = -2$
12. Particular solution: $y = -2x + \frac{1}{2}x^3$

37

Use integration to find a general solution of the differential equation.

$\frac{dy}{dx} = 3x^2$

1. $y = \int 3x^2\,dx$
2. $= x^3 + C$

39

Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = \frac{x}{1 + x^2}$$

1. $y = \int \frac{x}{1 + x^2}\,dx$

2. $= \frac{1}{2}\ln(1 + x^2) + C$

3. $(u = 1 + x^2, du = 2x\,dx)$

41.

Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = \frac{x - 2}{x}$$

1. $\frac{x - 2}{x} = 1 - \frac{2}{x}$

2. $y = \int \left[1 - \frac{2}{x}\right] dx$

3. $= x - 2\ln|x| + C$

4. $= x - \ln x^2 + C$

43.

Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = \sin 2x$$

1. $y = \int \sin 2x\,dx$

2. $= -\frac{1}{2}\cos 2x + C$

3. $(u = 2x, du = 2dx)$

45/

Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = x\sqrt{x-3}$$

1. Let $u = \sqrt{x-3}$, then $x = u^2 + 3$
2. and $dx = 2u\,du$.
3. $y = \int x\sqrt{x-3}\,dx$
4. $= \int (u^2 + 3)(u)(2u)\,du$
5. $= 2\int (u^4 + 3u^2)\,du$
6. $= 2\left(\frac{u^5}{5} + u^3\right) + C$
7. $= \frac{2}{5}(x-3)^{5/2} + 2(x-3)^{3/2} + C$

47.

Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = xe^{x^2}$$

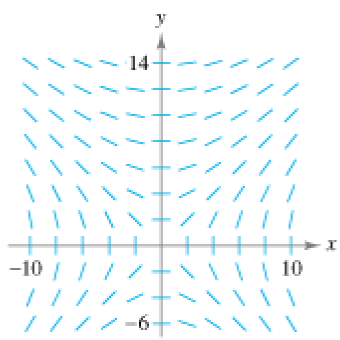
1. $y = \int xe^{x^2}\,dx$
2. $= \frac{1}{2}e^{x^2} + C$
3. $(u = x^2,\ du = 2x\,dx)$

49.

A differential equation and its slope field are given. Determine the slopes (if possible) in the slope field at the points given in the table.

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx						

$$\frac{dy}{dx} = \frac{x}{y}$$



1.

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx	-2	Undef.	0	$\frac{1}{2}$	$\frac{2}{3}$	1

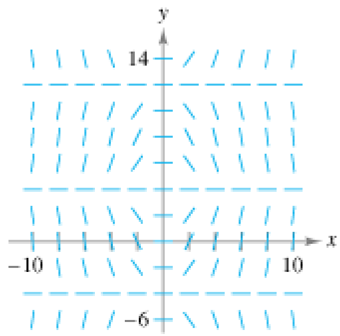
51.

A differential equation and its slope field are given. Determine the slopes (if possible) in the slope field at the points given in the table.

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx						

$$\frac{dy}{dx} = x \cos \frac{\pi y}{8}$$

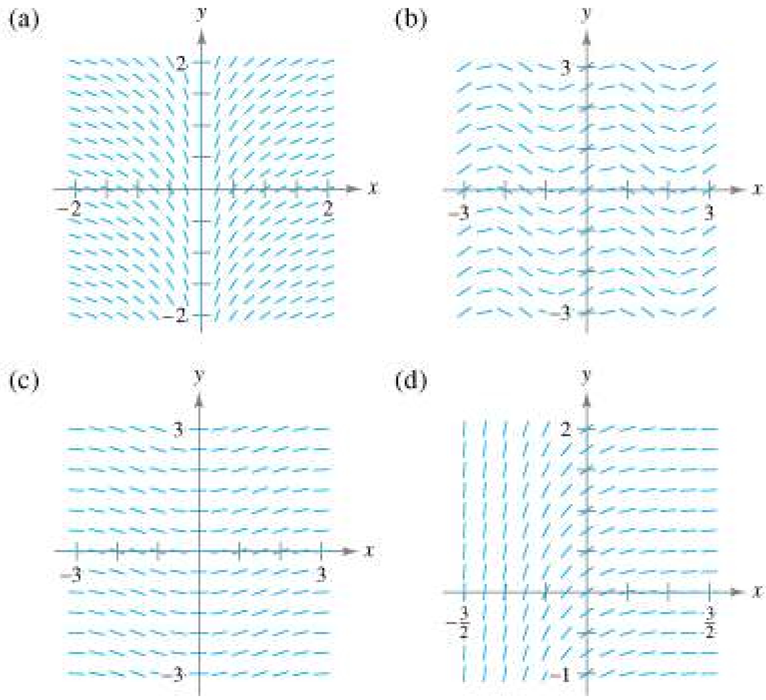
1.



x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx	$-2\sqrt{2}$	-2	0	0	$-2\sqrt{2}$	-8

53/

Match the differential equation with its slope field.
[The slope fields are labeled (a), (b), (c), and (d).]



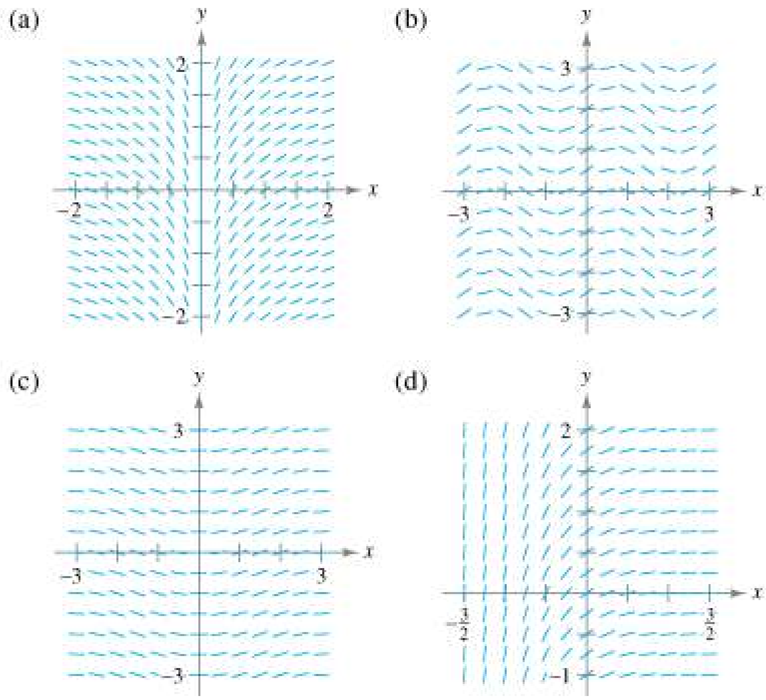
$$\frac{dy}{dx} = \cos(2x)$$

1. For $x = \pi$, $\frac{dy}{dx} = 1$.

2. Matches (b).

55.

Match the differential equation with its slope field.
[The slope fields are labeled (a), (b), (c), and (d).]



$$\frac{dy}{dx} = e^{-2x}$$

1. As $x \to \infty$, $\dfrac{dy}{dx} \to 0$.

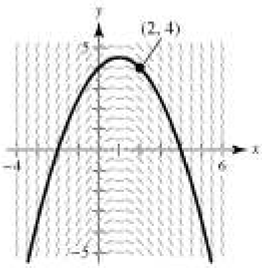
2. Matches (d).

57.

(a) Sketch the slope field for the differential equation, (b) use the slope field to sketch the solution that passes through the given point, and (c) discuss the graph of the solution as $x \to \infty$ and $x \to -\infty$.

Differential Equation	*Point*
$y' = -x + 1$	$(2, 4)$

1. (a), (b)



2. (c) As $x \to \infty$, $y \to -\infty$.

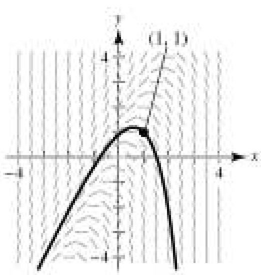
3. As $x \to -\infty$, $y \to -\infty$.

59.

(a) Sketch the slope field for the differential equation, (b) use the slope field to sketch the solution that passes through the given point, and (c) discuss the graph of the solution as $x \to \infty$ and $x \to -\infty$.

Differential Equation	*Point*
$y' = y - 2x$	$(1, 1)$

1. (a), (b)

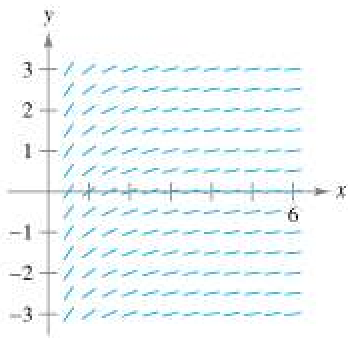


2. (c) As $x \to \infty$, $y \to -\infty$.

3. As $x \to -\infty$, $y \to -\infty$.

61

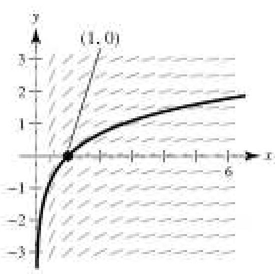
Use the slope field for the differential equation $y' = 1/x$, where $x > 0$, to sketch the graph of the solution that satisfies each given initial condition. Then make a conjecture about the behavior of a particular solution of $y' = 1/x$ as $x \to \infty$.



(a) $(1, 0)$ (b) $(2, -1)$

1. (a) $y' = \dfrac{1}{x}$, $y(1) = 0$

2.

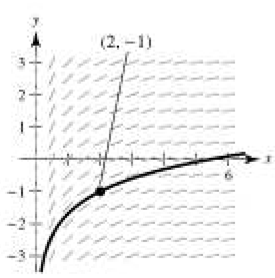


3. As $x \to \infty$, $y \to \infty$.

4. [**Note:** The solution is $y = \ln x$.]

5. (b) $y' = \dfrac{1}{x}$, $y(2) = -1$

6.

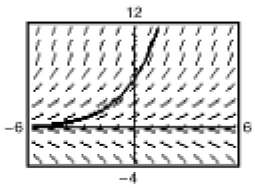


7. As $x \to \infty$, $y \to \infty$.

Use a computer algebra system to (a) graph the slope field for the differential equation and (b) graph the solution satisfying the specified initial condition.

$$\frac{dy}{dx} = 0.5y, \quad y(0) = 6$$

1.

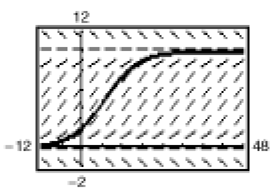


65.

Use a computer algebra system to (a) graph the slope field for the differential equation and (b) graph the solution satisfying the specified initial condition.

$$\frac{dy}{dx} = 0.02y(10 - y), \quad y(0) = 2$$

1.

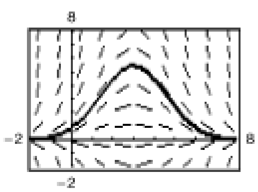


67

Use a computer algebra system to (a) graph the slope field for the differential equation and (b) graph the solution satisfying the specified initial condition.

$$\frac{dy}{dx} = 0.4y(3 - x), \quad y(0) = 1$$

1.



69

Use Euler's Method to make a table of values for the approximate solution of the differential equation with the specified initial value. Use n steps of size h.

$y' = x + y, \quad y(0) = 2, \quad n = 10, \quad h = 0.1$

1. $y_1 = y_0 + hF(x_0, y_0)$
2. $\quad = 2 + (0.1)(0 + 2)$
3. $\quad = 2.2$
4. $y_2 = y_1 + hF(x_1, y_1)$
5. $\quad = 2.2 + (0.1)(0.1 + 2.2)$
6. $\quad = 2.43$, etc.
7.

n	0	1	2	3	4	5	6	7	8	9	10
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y_n	2	2.2	2.43	2.693	2.992	3.332	3.715	4.146	4.631	5.174	5.781

71.

Use Euler's Method to make a table of values for the approximate solution of the differential equation with the specified initial value. Use n steps of size h.

$y' = 3x - 2y, \; y(0) = 3, \; n = 10, \; h = 0.05$

1. $y_1 = y_0 + hF(x_0, y_0)$
2. $= 3 + (0.05)(3(0) - 2(3))$
3. $= 2.7$
4. $y_2 = y_1 + hF(x_1, y_1)$
5. $= 2.7 + (0.05)(3(0.05) - 2(2.7))$
6. $= 2.4375$, etc.
7.

n	0	1	2	3	4	5	6	7	8	9	10
x_n	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
y_n	3	2.7	2.438	2.209	2.010	1.839	1.693	1.569	1.464	1.378	1.308

73

Use Euler's Method to make a table of values for the approximate solution of the differential equation with the specified initial value. Use n steps of size h.

$y' = e^{xy}, \; y(0) = 1, \; n = 10, \; h = 0.1$

1. $y_1 = y_0 + hF(x_0, y_0)$
2. $= 1 + (0.1)e^{0(1)}$
3. $= 1.1$
4. $y_2 = y_1 + hF(x_1, y_1)$
5. $= 1.1 + (0.1)e^{(0.1)(1.1)}$
6. ≈ 1.2116, etc.
7.

n	0	1	2	3	4	5	6	7	8	9	10
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y_n	1	1.1	1.212	1.339	1.488	1.670	1.900	2.213	2.684	3.540	5.958

Complete the table using the exact solution of the differential equation and two approximations obtained using Euler's Method to approximate the particular solution of the differential equation. Use $h = 0.2$ and 0.1 and compute each approximation to four decimal places.

x	0	0.2	0.4	0.6	0.8	1.0
$y(x)$ (exact)						
$y(x)$ $(h = 0.2)$						
$y(x)$ $(h = 0.1)$						

Differential Equation	*Initial Condition*	*Exact Solution*
$\frac{dy}{dx} = y$	$(0, 3)$	$y = 3e^x$

1.

x	0	0.2	0.4	0.6	0.8	1.0
$y(x)$ (exact)	3	3.6642	4.4755	5.4664	6.6766	8.1548
$y(x)$ $(h = 0.2)$	3	3.6000	4.3200	5.1840	6.2208	7.4650
$y(x)$ $(h = 0.1)$	3	3.6300	4.3923	5.3147	6.4308	7.7812

75.

Complete the table using the exact solution of the differential equation and two approximations obtained using Euler's Method to approximate the particular solution of the differential equation. Use $h = 0.2$ and 0.1 and compute each approximation to four decimal places.

x	0	0.2	0.4	0.6	0.8	1.0
$y(x)$ (exact)						
$y(x)$ $(h = 0.2)$						
$y(x)$ $(h = 0.1)$						

Differential Equation	*Initial Condition*	*Exact Solution*
$\frac{dy}{dx} = y$	(0, 3)	$y = 3e^x$

1.

x	0	0.2	0.4	0.6	0.8	1.0
$y(x)$ (exact)	3	3.6642	4.4755	5.4664	6.6766	8.1548
$y(x)$ $(h = 0.2)$	3	3.6000	4.3200	5.1840	6.2208	7.4650
$y(x)$ $(h = 0.1)$	3	3.6300	4.3923	5.3147	6.4308	7.7812

Complete the table using the exact solution of the differential equation and two approximations obtained using Euler's Method to approximate the particular solution of the differential equation. Use $h = 0.2$ and 0.1 and compute each approximation to four decimal places.

x	0	0.2	0.4	0.6	0.8	1.0
$y(x)$ (exact)						
$y(x)$ $(h = 0.2)$						
$y(x)$ $(h = 0.1)$						

Differential Equation	*Initial Condition*	*Exact Solution*
$\frac{dy}{dx} = y + \cos(x)$	$(0, 0)$	$y = \frac{1}{2}(\sin x - \cos x + e^x)$

1.

x	0	0.2	0.4	0.6	0.8	1.0
$y(x)$ (exact)	0	0.2200	0.4801	0.7807	1.1231	1.5097
$y(x)$ $(h = 0.2)$	0	0.2000	0.4360	0.7074	1.0140	1.3561
$y(x)$ $(h = 0.1)$	0	0.2095	0.4568	0.7418	1.0649	1.4273

At time $t = 0$ minutes, the temperature of an object is 140°F. The temperature of the object is changing at the rate given by the differential equation

$$\frac{dy}{dt} = -\frac{1}{2}(y - 72).$$

(a) Use a graphing utility and Euler's Method to approximate the particular solutions of this differential equation at $t = 1, 2,$ and 3. Use a step size of $h = 0.1$. (A graphing utility program for Euler's Method is available on the website *college.hmco.com.*)

(b) Compare your results with the exact solution $y = 72 + 68e^{-t/2}$.

1. $y(0) = 140, \ h = 0.1$

2. (a)

t	0	1	2	3
Euler	140	112.7	96.4	86.6

3. (b)

t	0	1	2	3
Exact	140	113.24	97.016	87.173

81.

In your own words, describe the difference between a general solution of a differential equation and a particular solution.

1. A general solution of order n has n arbitrary constants while in a particular solution initial conditions are given in order to solve for all these constants.

83.

Describe how to use Euler's Method to approximate the particular solution of a differential equation.

1. Consider $y' = F(x, y)$,
2. $y(x_0) = y_0$.
3. Begin with a point (x_0, y_0) that satisfies the initial condition, $y(x_0) = y_0$.
4. Then using a step size of h, find the point $(x_1, y_1) = (x_0 + h, y_0 + hF(x_0, y_0))$.
5. Continue generating the sequence of points $(x_{n+1}, y_{n+1}) = (x_n + h, y_n + hF(x_n, y_n))$.

85.

Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

If $y = f(x)$ is a solution of a first-order differential equation, then $y = f(x) + C$ is also a solution.

1. False
2. Consider Example 2.
3. $y = x^3$ is a solution to $xy' - 3y = 0$, but
4. $y = x^3 + 1$ is not a solution.

87.

Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

Slope fields represent the general solutions of differential equations.

1. True

89

The exact solution of the differential equation

$$\frac{dy}{dx} = -2y$$

where $y(0) = 4$, is $y = 4e^{-2x}$.

(a) Use a graphing utility to complete the table, where y is the exact value of the solution, y_1 is the approximate solution using Euler's Method with $h = 0.1$, y_2 is the approximate solution using Euler's Method with $h = 0.2$, e_1 is the absolute error $|y - y_1|$, e_2 is the absolute error $|y - y_2|$, and r is the ratio e_1/e_2.

x	0	0.2	0.4	0.6	0.8	1.0
y						
y_1						
y_2						
e_1						
e_2						
r						

(b) What can you conclude about the ratio r as h changes?

(c) Predict the absolute error when $h = 0.05$.

1. (a)

x	0	0.2	0.4	0.6	0.8	1.0
y	4	2.6813	1.7973	1.2048	0.8076	0.5413
y_1	4	2.5600	1.6384	1.0486	0.6711	0.4295
y_2	4	2.4000	1.4400	0.8640	0.5184	0.3110
e_1	0	0.1213	0.1589	0.1562	0.1365	0.1118
e_2	0	0.2813	0.3573	0.3408	0.2892	0.2303
r		0.4312	0.4447	0.4583	0.4720	0.4855

2. (b) If h is halved, then the error is approximately halved ($r \approx 0.5$).

3. (c) When $h = 0.05$, the errors will again be approximately halved.

It is known that $y = A \sin \omega t$ is a solution of the differential equation $y'' + 16y = 0$. Find the values of ω.

1. $$y' = A\omega \cos \omega t$$
2. $$y'' = -A\omega^2 \sin \omega t$$
3. $$y'' + 16y = 0$$
4. $-A\omega^2 \sin \omega t + 16A \sin \omega t = 0$
5. $$A \sin \omega t[16 - \omega^2] = 0$$
6. If $A \neq 0$, then $\omega = \pm 4$ radians/sec.

95

Prove that if the family of integral curves of the differential equation

$$\frac{dy}{dx} + p(x)y = q(x), \quad p(x) \cdot q(x) \neq 0$$

is cut by the line $x = k$, the tangents at the points of intersection are concurrent.

1. Let the vertical line $x = k$ cut the graph of the solution $y = f(x)$ at (k, t).
2. The tangent line at (k, t) is $y - t = f'(k)(x - k)$.
3. Since $y' + p(x)y = q(x)$, we have
4. $y - t = [q(k) - p(k)t](x - k)$.
5. For any value of t, this line passes through the point
$$\left(k + \frac{1}{p(k)}, \frac{q(k)}{p(k)}\right).$$
6. To see this, note that
$$\frac{q(k)}{p(k)} - t \stackrel{?}{=} [q(k) - p(k)t]\left(k + \frac{1}{p(k)} - k\right)$$
7. $$\stackrel{?}{=} q(k)k - p(k)tk + \frac{q(k)}{p(k)} - t - kq(k) + p(k)kt$$
8. $$= \frac{q(k)}{p(k)} - t.$$