Exercises for Section 6.2

The symbol [graphing icon] indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system.

Click on [S] to view the complete solution of the exercise.

Click on [M] to print an enlarged copy of the graph.

In Exercises 1–10, solve the differential equation.

[S] **1.** $\frac{dy}{dx} = x + 2$ **2.** $\frac{dy}{dx} = 4 - x$

[S] **3.** $\frac{dy}{dx} = y + 2$ **4.** $\frac{dy}{dx} = 4 - y$

[S] **5.** $y' = \frac{5x}{y}$ **6.** $y' = \frac{\sqrt{x}}{3y}$

[S] **7.** $y' = \sqrt{x}\,y$ **8.** $y' = x(1 + y)$

[S] **9.** $(1 + x^2)y' - 2xy = 0$ **10.** $xy + y' = 100x$

In Exercises 11–14, write and solve the differential equation that models the verbal statement.

[S] **11.** The rate of change of Q with respect to t is inversely proportional to the square of t.

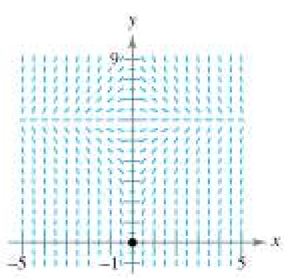
12. The rate of change of P with respect to t is proportional to $10 - t$.

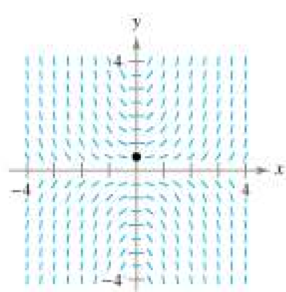
[S] **13.** The rate of change of N with respect to s is proportional to $250 - s$.

14. The rate of change of y with respect to x varies jointly as x and $L - y$.

[graphing icon] ***Slope Fields*** **In Exercises 15 and 16, a differential equation, a point, and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the given point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketch in part (a). To print an enlarged copy of the graph, select the MathGraph button.**

[M] [S] **15.** $\frac{dy}{dx} = x(6 - y)$, $(0, 0)$ [M] **16.** $\frac{dy}{dx} = xy$, $\left(0, \frac{1}{2}\right)$





[graphing icon] **In Exercises 17–20, find the function $y = f(t)$ passing through the point (0, 10) with the given first derivative. Use a graphing utility to graph the solution.**

[S] **17.** $\frac{dy}{dt} = \frac{1}{2}t$ **18.** $\frac{dy}{dt} = -\frac{3}{4}\sqrt{t}$

In Exercises 21–24, write and solve the di… that models the verbal statement. Evaluate … specified value of the independent variable.

[S] **21.** The rate of change of y is proportional to y. … and when $x = 3$, $y = 10$. What is the valu…

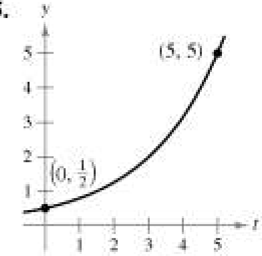
22. The rate of change of N is proportional … $N = 250$ and when $t = 1$, $N = 400$. Wha… when $t = 4$?

[S] **23.** The rate of change of V is proportional … $V = 20{,}000$ and when $t = 4$, $V = 12{,}500$. … V when $t = 6$?

24. The rate of change of P is proportional … $P = 5000$ and when $t = 1$, $P = 4750$. Wh… when $t = 5$?

In Exercises 25–28, find the exponential fun… passes through the two given points.

[S] **25.**



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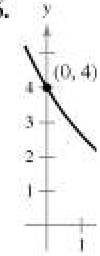
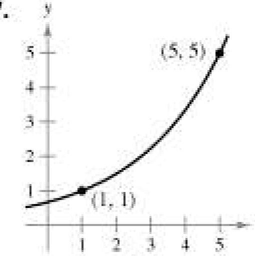


[S] **27.**



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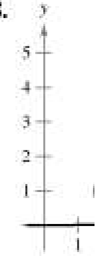

Writing About Concepts

[S] **29.** Describe what the values of C and k repr… nential growth and decay model, $y = Ce$…

30. Give the differential equation that m… growth and decay.

In Exercises 31 and 32, determine the qu… the solution of the differential equation … function. Explain. (Do not solve the differe…

[S] **31.** $\frac{dy}{dx} = \frac{1}{2}xy$ **32.** $\frac{dy}{dx} = \frac{1}{2}x^2$…

Radioactive Decay **In Exercises 33–40, complete the table for the radioactive isotope.**

	Isotope	*Half-Life (in years)*	*Initial Quantity*	*Amount After 1000 Years*	*Amount After 10,000 Years*
S **33.**	^{226}Ra	1599	10 g		
34.	^{226}Ra	1599		1.5 g	
S **35.**	^{226}Ra	1599			0.5 g
36.	^{14}C	5715			2 g
S **37.**	^{14}C	5715	5 g		
38.	^{14}C	5715		3.2 g	
S **39.**	^{239}Pu	24,100		2.1 g	
40.	^{239}Pu	24,100			0.4 g

S **41.** ***Radioactive Decay*** Radioactive radium has a half-life of approximately 1599 years. What percent of a given amount remains after 100 years?

42. ***Carbon Dating*** Carbon-14 dating assumes that the carbon dioxide on Earth today has the same radioactive content as it did centuries ago. If this is true, the amount of ^{14}C absorbed by a tree that grew several centuries ago should be the same as the amount of ^{14}C absorbed by a tree growing today. A piece of ancient charcoal contains only 15% as much of the radioactive carbon as a piece of modern charcoal. How long ago was the tree burned to make the ancient charcoal? (The half-life of ^{14}C is 5715 years.)

Compound Interest **In Exercises 43–48, complete the table for a savings account in which interest is compounded continuously.**

	Initial Investment	*Annual Rate*	*Time to Double*	*Amount After 10 Years*
S **43.**	$1000	6%		
44.	$20,000	$5\frac{1}{2}\%$		
S **45.**	$750		$7\frac{3}{4}$ yr	
46.	$10,000		5 yr	
S **47.**	$500			$1292.85
48.	$2000			$5436.56

Compound Interest **In Exercises 49–52, find the principal P that must be invested at rate r, compounded monthly, so that $500,000 will be available for retirement in t years.**

S **49.** $r = 7\frac{1}{2}\%,\quad t = 20$

50. $r = 6\%,\quad t = 40$

S **51.** $r = 8\%,\quad t = 35$

52. $r = 9\%,\quad t = 25$

Compound Interest **In Exercises 53–56, find the time necessary for $1000 to double if it is invested at a rate of r compounded (a) annually, (b) monthly, (c) daily, and (d) continuously.**

S **53.** $r = 7\%$

54. $r = 6\%$

S **55.** $r = 8.5\%$

56. $r = 5.5\%$

Population **In Exercises 57–60, the populat a country in 2001 and the expected continu change k of the population for the years 200 given. (*Source: U.S. Census Bureau, Intern***

(a) Find the exponential growth model P = lation by letting $t = 0$ correspond to 200

(b) Use the model to predict the populatior 2015.

(c) Discuss the relationship between the s change in population for the country.

	Country	*2001 Population*
S **57.**	Bulgaria	7.7
58.	Cambodia	12.7
S **59.**	Jordan	5.2
60.	Lithuania	3.6

S **61.** ***Modeling Data*** One hundred bacteria ar and the number N of bacteria is counted ea The results are shown in the table, where t

t	0	1	2	3	4	5
N	100	126	151	198	243	29

(a) Use the regression capabilities of a gra an exponential model for the data.

(b) Use the model to estimate the time req tion to quadruple in size.

62. ***Bacteria Growth*** The number of bacte increasing according to the law of expone are 125 bacteria in the culture after 2 hou after 4 hours.

(a) Find the initial population.

(b) Write an exponential growth model fo lation. Let t represent time in hours.

(c) Use the model to determine the numb 8 hours.

(d) After how many hours will the bacteri

S **63.** ***Learning Curve*** The management at a found that a worker can produce at most 3 learning curve for the number of units N pr a new employee has worked t days is N = days on the job, a particular worker produc

(a) Find the learning curve for this worker.

(b) How many days should pass before this 25 units per day?

64. ***Learning Curve*** If in Exercise 63 man new employee to produce at least 20 units p on the job, find (a) the learning curve minimum requirement and (b) the numb minimal achiever is producing 25 units per

65. ***Modeling Data*** The table shows the population P (in millions) of the United States from 1960 to 2000. *(Source: U.S. Census Bureau)*

Year	1960	1970	1980	1990	2000
Population, P	181	205	228	250	282

(a) Use the 1960 and 1970 data to find an exponential model P_1 for the data. Let $t = 0$ represent 1960.

(b) Use a graphing utility to find an exponential model P_2 for the data. Let $t = 0$ represent 1960.

(c) Use a graphing utility to plot the data and graph both models in the same viewing window. Compare the actual data with the predictions. Which model better fits the data?

(d) Estimate when the population will be 320 million.

66. ***Modeling Data*** The table shows the net receipts and the amounts required to service the national debt (interest on Treasury debt securities) of the United States from 1992 through 2001. The monetary amounts are given in billions of dollars. *(Source: U.S. Office of Management and Budget)*

Year	1992	1993	1994	1995	1996
Receipts	1091.3	1154.4	1258.6	1351.8	1453.1
Interest	292.3	292.5	296.3	332.4	343.9

Year	1997	1998	1999	2000	2001
Receipts	1579.3	1721.8	1827.5	2025.2	1991.2
Interest	355.8	363.8	353.5	361.9	359.5

(a) Use the regression capabilities of a graphing utility to find an exponential model R for the receipts and a quartic model I for the amount required to service the debt. Let t represent the time in years, with $t = 2$ corresponding to 1992.

(b) Use a graphing utility to plot the points corresponding to the receipts, and graph the corresponding model. Based on the model, what is the continuous rate of growth of the receipts?

(c) Use a graphing utility to plot the points corresponding to the amount required to service the debt, and graph the quartic model.

(d) Find a function $P(t)$ that approximates the percent of the receipts that is required to service the national debt. Use a graphing utility to graph this function.

67. ***Sound Intensity*** The level of sound β (in decibels), with an intensity of I is

$$\beta(I) = 10 \log_{10} \frac{I}{I_0}$$

where I_0 is an intensity of 10^{-16} watts per square centimeter, corresponding roughly to the faintest sound that can be heard. Determine $\beta(I)$ for the following.

(a) $I = 10^{-14}$ watts per square centimeter (whisper)

(b) $I = 10^{-9}$ watts per square centimeter (

(c) $I = 10^{-6.5}$ watts per square centimeter

(d) $I = 10^{-4}$ watts per square centimeter (

68. ***Noise Level*** With the installation of materials, the noise level in an auditorium 93 to 80 decibels. Use the function in Ex percent decrease in the intensity level of th the installation of these materials.

69. ***Forestry*** The value of a tract of timber is

$$V(t) = 100{,}000e^{0.8\sqrt{t}}$$

where t is the time in years, with $t = 0$ cor If money earns interest continuously at 10 of the timber at any time t is $A(t) = V(t)e^{-}$ which the timber should be harvested to m value function.

70. ***Earthquake Intensity*** On the Richter sca of an earthquake of intensity I is

$$R = \frac{\ln I - \ln I_0}{\ln 10}$$

where I_0 is the minimum intensity us Assume that $I_0 = 1$.

(a) Find the intensity of the 1906 San F ($R = 8.3$).

(b) Find the factor by which the intensity Richter scale measurement is doubled.

(c) Find dR/dI.

71. ***Newton's Law of Cooling*** When an obj a furnace and placed in an environme temperature of 80°F, its core temperature i after it is removed, the core temperature core temperature 5 hours after the object i furnace.

72. ***Newton's Law of Cooling*** A container of in a freezer that is kept at a constant temp initial temperature of the liquid is 160°F. liquid's temperature is 60°F. How much lo its temperature to decrease to 30°F?

True or False? **In Exercises 73–76, deter statement is true or false. If it is false, expl example that shows it is false.**

73. In exponential growth, the rate of growth i

74. In linear growth, the rate of growth is cons

75. If prices are rising at a rate of 0.5% per m rising at a rate of 6% per year.

76. The differential equation modeling exp $dy/dx = ky$, where k is a constant.