1

Solve the differential equation.

$$\frac{dy}{dx} = x + 2$$

1. $y = \int (x + 2)\, dx$

2. $= \frac{x^2}{2} + 2x + C$

3

Solve the differential equation.

$$\frac{dy}{dx} = y + 2$$

1. $\frac{dy}{y + 2} = dx$

2. $\int \frac{1}{y + 2}\, dy = \int dx$

3. $\ln|y + 2| = x + C_1$

4. $y + 2 = e^{x + C_1}$

5. $= Ce^x$

6. $y = Ce^x - 2$

5

Solve the differential equation.

$$y' = \frac{5x}{y}$$

1. $yy' = 5x$
2. $\int yy'\,dx = \int 5x\,dx$
3. $\int y\,dy = \int 5x\,dx$
4. $\frac{1}{2}y^2 = \frac{5}{2}x^2 + C_1$
5. $y^2 - 5x^2 = C$

7

Solve the differential equation.

$$y' = \sqrt{x}\,y$$

1. $\frac{y'}{y} = \sqrt{x}$
2. $\int \frac{y'}{y}\,dx = \int \sqrt{x}\,dx$
3. $\int \frac{dy}{y} = \int \sqrt{x}\,dx$
4. $\ln|y| = \frac{2}{3}x^{3/2} + C_1$
5. $y = e^{(2/3)x^{3/2} + C_1}$
6. $= e^{C_1}e^{(2/3)x^{3/2}}$
7. $= Ce^{(2/3)x^{3/2}}$

9

Solve the differential equation.

$(1 + x^2)y' - 2xy = 0$

1. $y' = \dfrac{2xy}{1 + x^2}$

2. $\dfrac{y'}{y} = \dfrac{2x}{1 + x^2}$

3. $\displaystyle\int \frac{y'}{y}\,dx = \int \frac{2x}{1 + x^2}\,dx$

4. $\displaystyle\int \frac{dy}{y} = \int \frac{2x}{1 + x^2}\,dx$

5. $\ln|y| = \ln(1 + x^2) + C_1$

6. $\ln|y| = \ln(1 + x^2) + \ln C$

7. $\ln|y| = \ln[C(1 + x^2)]$

8. $y = C(1 + x^2)$

11

Write and solve the differential equation that models the verbal statement.

The rate of change of Q with respect to t is inversely proportional to the square of t.

1. $\dfrac{dQ}{dt} = \dfrac{k}{t^2}$

2. $\displaystyle\int \frac{dQ}{dt}\,dt = \int \frac{k}{t^2}\,dt$

3. $\displaystyle\int dQ = -\frac{k}{t} + C$

4. $Q = -\dfrac{k}{t} + C$

13

Write and solve the differential equation that models the verbal statement.

The rate of change of N with respect to s is proportional to $250 - s$.

1. $\frac{dN}{ds} = k(250 - s)$

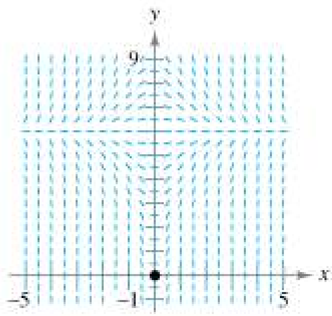
2. $\int \frac{dN}{ds}\, ds = \int k(250 - s)\, ds$

3. $\int dN = -\frac{k}{2}(250 - s)^2 + C$

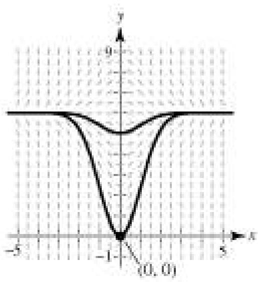
4. $N = -\frac{k}{2}(250 - s)^2 + C$

A differential equation, a point, and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the given point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketch in part (a).

$$\frac{dy}{dx} = x(6 - y), \quad (0, 0)$$



1. (a)



2. (b) $\displaystyle \frac{dy}{y-6} = -x\,dx$

3. $\displaystyle \ln|y - 6| = \frac{-x^2}{2} + C$

4. $y - 6 = e^{-x^2/2 + C}$

5. $= C_1 e^{-x^2/2}$

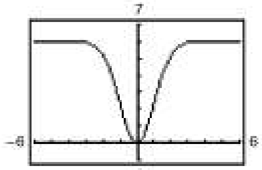
6. $y = 6 + C_1 e^{-x^2/2}$

7. $(0, 0)$: $0 = 6 + C_1$

8. $\Rightarrow C_1 = -6$

9. $\Rightarrow y = 6 - 6e^{-x^2/2}$

10.



17

Find the function $y = f(t)$ passing through the point $(0, 10)$ with the given first derivative. Use a graphing utility to graph the solution.

$$\frac{dy}{dt} = \frac{1}{2}t$$

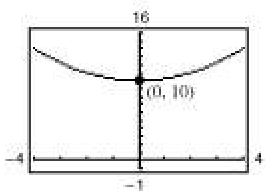
1. $\int dy = \int \frac{1}{2}t \, dt$

2. $y = \frac{1}{4}t^2 + C$

3. $10 = \frac{1}{4}(0)^2 + C$

4. $\Rightarrow C = 10$

5. $y = \frac{1}{4}t^2 + 10$

6. 

19

Find the function $y = f(t)$ passing through the point $(0, 10)$ with the given first derivative. Use a graphing utility to graph the solution.

$$\frac{dy}{dt} = -\frac{1}{2}y$$

1. $\int \frac{dy}{y} = \int -\frac{1}{2}\,dt$

2. $\ln|y| = -\frac{1}{2}t + C_1$

3. $y = e^{-(t/2) + C_1}$

4. $= e^{C_1} e^{-t/2}$

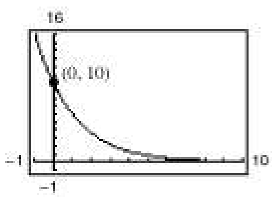
5. $= Ce^{-t/2}$

6. $10 = Ce^0$

7. $\Rightarrow C = 10$

8. $y = 10e^{-t/2}$

9.



Write and solve the differential equation that models the verbal statement. Evaluate the solution at the specified value of the independent variable.

The rate of change of y is proportional to y. When $x = 0$, $y = 4$ and when $x = 3$, $y = 10$. What is the value of y when $x = 6$?

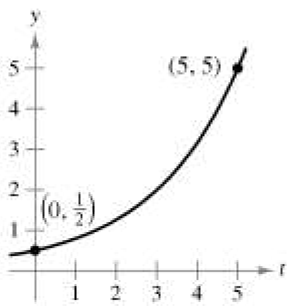
1. $\frac{dy}{dx} = ky$
2. $y = Ce^{kx}$ (Theorem 5.16)
3. $(0, 4)$: $4 = Ce^{0}$
4. $= C$
5. $(3, 10)$: $10 = 4e^{3k}$
6. $\Longrightarrow k = \frac{1}{3}\ln\left(\frac{5}{2}\right)$
7. When $x = 6$:
8. $y = 4e^{1/3\ln(5/2)(6)}$
9. $= 4e^{\ln(5/2)^2}$
10. $= 4\left(\frac{5}{2}\right)^2$
11. $= 25$

Write and solve the differential equation that models the verbal statement. Evaluate the solution at the specified value of the independent variable.

The rate of change of V is proportional to V. When $t = 0$, $V = 20{,}000$ and when $t = 4$, $V = 12{,}500$. What is the value of V when $t = 6$?

1. $\dfrac{dV}{dt} = kV$
2. $V = Ce^{kt}$ (Theorem 5.16)
3. $(0, 20{,}000)$: $C = 20{,}000$
4. $(4, 12{,}500)$: $12{,}500 = 20{,}000e^{4k}$
5. $\Rightarrow k = \dfrac{1}{4}\ln\left(\dfrac{5}{8}\right)$
6. When $t = 6$:
7. $V = 20{,}000e^{1/4 \ln(5/8)(6)}$
8. $= 20{,}000e^{\ln(5/8)^{3/2}}$
9. $= 20{,}000\left(\dfrac{5}{8}\right)^{3/2}$
10. ≈ 9882.118

Find the exponential function $y = Ce^{kt}$ that passes through the two given points.



1. $C = \frac{1}{2}$

2. $y = \frac{1}{2}e^{kt}$

3. $5 = \frac{1}{2}e^{5k}$

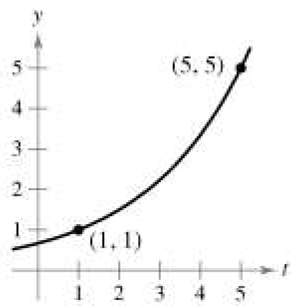
4. $k = \frac{\ln 10}{5}$

5. $y = \frac{1}{2}e^{(\ln 10/5)t}$

6. $= \frac{1}{2}(10^{t/5})$

7. or $y \approx \frac{1}{2}e^{0.4605t}$

Find the exponential function $y = Ce^{kt}$ that passes through the two given points.



1. $1 = Ce^{k}$
2. $5 = Ce^{5k}$
3. $5Ce^{k} = Ce^{5k}$
4. $5e^{k} = e^{5k}$
5. $5 = e^{4k}$
6. $k = \frac{\ln 5}{4}$
7. ≈ 0.4024
8. $y = Ce^{0.4024t}$
9. $1 = Ce^{0.4024}$
10. $C \approx 0.6687 \quad (C = 5^{-1/4})$
11. $y \approx 0.6687e^{0.4024t}$

29

Describe what the values of C and k represent in the exponential growth and decay model, $y = Ce^{kt}$.

1. In the model $y = Ce^{kt}$, C represents the initial value of y (when $t = 0$).
2. k is the proportionality constant.

31

Determine the quadrants in which the solution of the differential equation is an increasing function. Explain. (Do not solve the differential equation.)

$$\frac{dy}{dx} = \frac{1}{2}xy$$

1. $\frac{dy}{dx} > 0$ when $xy > 0$.
2. Quadrants I and III.

33

Complete the table for the radioactive isotope.

Isotope	*Half-Life (in years)*	*Initial Quantity*	*Amount After 1000 Years*	*Amount After 10,000 Years*
^{226}Ra	1599	10 g		

1. Since the initial quantity is 10 grams, $y = 10e^{kt}$.
2. Since the half-life is 1599 years

   $5 = 10e^{k(1599)}$
3. $k = \frac{1}{1599}\ln\left(\frac{1}{2}\right)$.
4. Thus, $y = 10e^{[\ln(1/2)/1599]t}$.
5. When $t = 1000$,

   $y = 10e^{[\ln(1/2)/1599](1000)}$
6. ≈ 6.48 g.
7. When $t = 10{,}000$, $y \approx 0.13$ g.

35

Complete the table for the radioactive isotope.

Isotope	*Half-Life (in years)*	*Initial Quantity*	*Amount After 1000 Years*	*Amount After 10,000 Years*
^{226}Ra	1599			0.5 g

1. Since the half-life is 1599 years,

$$\frac{1}{2} = 1e^{k(1599)}$$

2. $k = \frac{1}{1599}\ln\left(\frac{1}{2}\right)$.

3. Since there is 0.5 gram after 10,000 years

4. $0.5 = Ce^{[\ln(1/2)/1599](10,000)}$

5. $C \approx 38.158$.

6. Hence, the initial quantity is approximately 38.158 g.

7. When $t = 1000$,

$$y = 38.158e^{[\ln(1/2)/1599](1000)}$$

8. ≈ 24.74 g.

Complete the table for the radioactive isotope.

Isotope	*Half-Life (in years)*	*Initial Quantity*	*Amount After 1000 Years*	*Amount After 10,000 Years*
^{14}C	5715	5 g		

1. Since the initial quantity is 5 grams, $C = 5$.
2. Since the half-life is 5715 years

   $2.5 = 5e^{k(5715)}$
3. $k = \dfrac{1}{5715}\ln\left(\dfrac{1}{2}\right).$
4. When $t = 1000$ years,

   $y = 5e^{[\ln(1/2)/5715](1000)}$
5. ≈ 4.43 g.
6. When $t = 10{,}000$ years,

   $y = 5e^{[\ln(1/2)/5715](10{,}000)}$
7. ≈ 1.49 g.

Complete the table for the radioactive isotope.

Isotope	*Half-Life (in years)*	*Initial Quantity*	*Amount After 1000 Years*	*Amount After 10,000 Years*
^{239}Pu	24,100		2.1 g	

1. Since the half-life is 24,100 years,

   $\frac{1}{2} = 1e^{k(24,100)}$

2. $k = \frac{1}{24,100} \ln\left(\frac{1}{2}\right)$.

3. Since there are 2.1 grams after 1000 years

4. $2.1 = Ce^{[\ln(1/2)/24,100](1000)}$

5. $C \approx 2.161$.

6. Thus, the initial quantity is approximately 2.161 g.

7. When $t = 10{,}000$,

   $y = 2.161e^{[\ln(1/2)/24,100](10,000)}$

8. ≈ 1.62 g.

41

Radioactive radium has a half-life of approximately 1599 years. What percent of a given amount remains after 100 years?

1. $y = Ce^{kt}$

2. $\frac{1}{2}C = Ce^{k(1599)}$

3. $k = \frac{1}{1599} \ln\left(\frac{1}{2}\right)$

4. When $t = 100$,

   $y = Ce^{[\ln(1/2)/1599](100)}$

5. $\approx 0.9576\, C$

6. Therefore, 95.76% remains after 100 years.

43

Complete the table for a savings account in which interest is compounded continuously.

Initial Investment	*Annual Rate*	*Time to Double*	*Amount After 10 Years*
\$1000	6%		

1. Since $A = 1000e^{0.06t}$, the time to double is given by $2000 = 1000e^{0.06t}$ and we have
2. $2 = e^{0.06t}$
3. $\ln 2 = 0.06t$
4. $t = \dfrac{\ln 2}{0.06}$
5. ≈ 11.55 years.
6. Amount after 10 years:

   $A = 1000e^{(0.06)(10)}$
7. $\approx \$1822.12$

45

Complete the table for a savings account in which interest is compounded continuously.

Initial Investment	*Annual Rate*	*Time to Double*	*Amount After 10 Years*
\$750		$7\frac{3}{4}$ yr	

1. Since $A = 750e^{rt}$ and $A = 1500$ when $t = 7.75$, we have the following.
2. $1500 = 750e^{7.75r}$
3. $r = \dfrac{\ln 2}{7.75}$
4. ≈ 0.0894
5. $= 8.94\%$
6. Amount after 10 years:

   $A = 750e^{0.0894(10)}$
7. $\approx \$1833.67$

47

Complete the table for a savings account in which interest is compounded continuously.

Initial Investment	*Annual Rate*	*Time to Double*	*Amount After 10 Years*
\$500			\$1292.85

1. Since $A = 500e^{rt}$ and $A = 1292.85$ when $t = 10$, we have the following.
2. $1292.85 = 500e^{10r}$
3. $r = \dfrac{\ln(1292.85/500)}{10}$
4. ≈ 0.0950
5. $= 9.50\%$
6. The time to double is given by
   $1000 = 500e^{0.0950t}$
7. $t = \dfrac{\ln 2}{0.095}$
8. ≈ 7.30 years.

49

Find the principal P that must be invested at rate r, compounded monthly, so that \$500,000 will be available for retirement in t years.

$r = 7\frac{1}{2}\%,\ t = 20$

1. $500{,}000 = P\left(1 + \dfrac{0.075}{12}\right)^{(12)(20)}$
2. $P = 500{,}000\left(1 + \dfrac{0.075}{12}\right)^{-240}$
3. $\approx \$112{,}087.09$

51

Find the principal P that must be invested at rate r, compounded monthly, so that \$500,000 will be available for retirement in t years.

$r = 8\%, \ t = 35$

1. $500{,}000 = P\left(1 + \frac{0.08}{12}\right)^{(12)(35)}$

2. $P = 500{,}000\left(1 + \frac{0.08}{12}\right)^{-420}$

3. $= \$30{,}688.87$

Find the time necessary for \$1000 to double if it is invested at a rate of r compounded (a) annually, (b) monthly, (c) daily, and (d) continuously.

$r = 7\%$

1. (a) $2000 = 1000(1 + 0.07)^t$

2. $2 = 1.07^t$

3. $\ln 2 = t \ln 1.07$

4. $t = \dfrac{\ln 2}{\ln 1.07}$

5. ≈ 10.24 years

6. (b) $2000 = 1000\left(1 + \dfrac{0.07}{12}\right)^{12t}$

7. $2 = \left(1 + \dfrac{0.007}{12}\right)^{12t}$

8. $\ln 2 = 12t \ln\left(1 + \dfrac{0.07}{12}\right)$

9. $t = \dfrac{\ln 2}{12 \ln(1 + (0.07/12))}$

10. ≈ 9.93 years

11. (c) $2000 = 1000\left(1 + \dfrac{0.07}{365}\right)^{365t}$

12. $2 = \left(1 + \dfrac{0.07}{365}\right)^{365t}$

13. $\ln 2 = 365t \ln\left(1 + \dfrac{0.07}{365}\right)$

14. $t = \dfrac{\ln 2}{365 \ln(1 + (0.07/365))}$

15. ≈ 9.90 years

16. (d) $2000 = 1000e^{(0.07)t}$

17. $2 = e^{0.07t}$

18. $\ln 2 = 0.07t$

19. $t = \dfrac{\ln 2}{0.07}$

20. ≈ 9.90 years

Find the time necessary for \$1000 to double if it is invested at a rate of r compounded (a) annually, (b) monthly, (c) daily, and (d) continuously.

$r = 8.5\%$

1. (a) $2000 = 1000(1 + 0.085)^t$

2. $2 = 1.085^t$

3. $\ln 2 = t \ln 1.085$

4. $t = \dfrac{\ln 2}{\ln 1.085}$

5. ≈ 8.50 years

6. (b) $2000 = 1000\left(1 + \dfrac{0.085}{12}\right)^{12t}$

7. $2 = \left(1 + \dfrac{0.085}{12}\right)^{12t}$

8. $\ln 2 = 12t \ln\left(1 + \dfrac{0.085}{12}\right)$

9. $t = \dfrac{1}{12} \dfrac{\ln 2}{\ln\left(1 + \dfrac{0.085}{12}\right)}$

10. ≈ 8.18 years

11. (c) $2000 = 1000\left(1 + \dfrac{0.085}{365}\right)^{365t}$

12. $2 = \left(1 + \dfrac{0.085}{365}\right)^{365t}$

13. $\ln 2 = 365t \ln\left(1 + \dfrac{0.085}{365}\right)$

14. $t = \dfrac{1}{365} \dfrac{\ln 2}{\ln\left(1 + \dfrac{0.085}{365}\right)}$

15. ≈ 8.16 years

16. (d) $2000 = 1000e^{0.085t}$

17. $2 = e^{0.085t}$

18. $\ln 2 = 0.085t$

19. $t = \dfrac{\ln 2}{0.085}$

20. ≈ 8.15 years

57

The population (in millions) of a country in 2001 and the expected continuous annual rate of change k of the population for the years 2000 through 2010 are given. *(Source: U.S. Census Bureau, International Data Base)*

(a) Find the exponential growth model $P = Ce^{kt}$ for the population by letting $t = 0$ correspond to 2000.

(b) Use the model to predict the population of the country in 2015.

(c) Discuss the relationship between the sign of k and the change in population for the country.

Country	2001 *Population*	k
Bulgaria	7.7	-0.009

1. (a) $P = Ce^{kt}$
2. $= Ce^{-0.009t}$
3. $P(1) = 7.7$
4. $= Ce^{-0.009(1)}$
5. $C \approx 7.7696$
6. $P = 7.7696e^{-0.009t}$
7. (b) For $t = 15$, $P = 7.7696e^{-0.009(15)}$
8. ≈ 6.79 million.
9. (c) If $k < 0$, the population is decreasing.

59

The population (in millions) of a country in 2001 and the expected continuous annual rate of change k of the population for the years 2000 through 2010 are given. *(Source: U.S. Census Bureau, International Data Base)*

(a) Find the exponential growth model $P = Ce^{kt}$ for the population by letting $t = 0$ correspond to 2000.

(b) Use the model to predict the population of the country in 2015.

(c) Discuss the relationship between the sign of k and the change in population for the country.

Country	2001 *Population*	k
Jordan	5.2	0.026

1. (a) $P = Ce^{kt}$
2. $= Ce^{0.026t}$
3. $5.2 = P(1)$
4. $= Ce^{0.026(1)}$
5. $C \approx 5.0665$
6. $P = 5.0665e^{0.026t}$
7. (b) For $t = 15$, $P = 5.0665e^{0.026(15)}$
8. ≈ 7.48 million.
9. (c) For $k > 0$, the population is increasing.

One hundred bacteria are started in a culture and the number N of bacteria is counted each hour for 5 hours. The results are shown in the table, where t is the time in hours.

t	0	1	2	3	4	5
N	100	126	151	198	243	297

(a) Use the regression capabilities of a graphing utility to find an exponential model for the data.

(b) Use the model to estimate the time required for the population to quadruple in size.

1. (a) $N = 100.1596(1.2455)^t$

2. (b) $N = 400$ when $t = 6.3$ hours (graphing utility)

3. Analytically,

$$400 = 100.1596(1.2455)^t$$

4. $$1.2455^t = \frac{400}{100.1596}$$

5. $$= 3.9936$$

6. $$t \ln 1.2455 = \ln 3.9936$$

7. $$t = \frac{\ln 3.9936}{\ln 1.2455}$$

8. $$\approx 6.3 \text{ hours.}$$

The management at a certain factory has found that a worker can produce at most 30 units in a day. The learning curve for the number of units N produced per day after a new employee has worked t days is $N = 30(1 - e^{kt})$. After 20 days on the job, a particular worker produces 19 units.

(a) Find the learning curve for this worker.

(b) How many days should pass before this worker is producing 25 units a day?

1. (a) $19 = 30(1 - e^{20k})$

2. $30e^{20k} = 11$

3. $k = \frac{\ln(11/30)}{20}$

4. ≈ -0.0502

5. $N \approx 30(1 - e^{-0.0502t})$

6. (b) $25 = 30(1 - e^{-0.0502t})$

7. $e^{-0.0502t} = \frac{1}{6}$

8. $t = \frac{-\ln 6}{-0.0502}$

9. ≈ 36 days

The table shows the population P (in millions) of the United States from 1960 to 2000. *(Source: U.S. Census Bureau)*

Year	1960	1970	1980	1990	2000
Population, P	181	205	228	250	282

(a) Use the 1960 and 1970 data to find an exponential model P_1 for the data. Let $t = 0$ represent 1960.

(b) Use a graphing utility to find an exponential model P_2 for the data. Let $t = 0$ represent 1960.

(c) Use a graphing utility to plot the data and graph both models in the same viewing window. Compare the actual data with the predictions. Which model better fits the data?

(d) Estimate when the population will be 320 million.

1. (a) $P_1 = Ce^{kt}$

2. $= 181e^{kt}$

3. $205 = 181e^{10k}$

4. $\Rightarrow k = \frac{1}{10}\ln\left(\frac{205}{181}\right)$

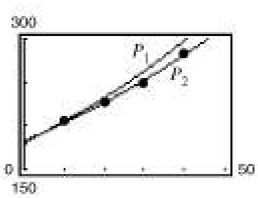
5. ≈ 0.01245

6. $P_1 \approx 181e^{0.01245t}$

7. $\approx 181(1.01253)^t$

8. (b) Using a graphing utility, $P_2 \approx 182.3248(1.01091)^t$

9. (c)



10. The model P_2 fits the data better.

11. (d) Using the model P_2,

$$320 = 182.3248(1.01091)^t$$

12. $\frac{320}{182.3248} = (1.01091)^t$

13. $t = \frac{\ln(320/182.3248)}{\ln(1.01091)}$

14. ≈ 51.8 years, or 2011.

67

The level of sound β (in decibels), with an intensity of I is

$$\beta(I) = 10 \log_{10} \frac{I}{I_0}$$

where I_0 is an intensity of 10^{-16} watts per square centimeter, corresponding roughly to the faintest sound that can be heard. Determine $\beta(I)$ for the following.

(a) $I = 10^{-14}$ watts per square centimeter (whisper)

(b) $I = 10^{-9}$ watts per square centimeter (busy street corner)

(c) $I = 10^{-6.5}$ watts per square centimeter (air hammer)

(d) $I = 10^{-4}$ watts per square centimeter (threshold of pain)

1. (a) $\beta(10^{-14}) = 10 \log_{10} \frac{10^{-14}}{10^{-16}}$

2. $= 20$ decibels

3. (b) $\beta(10^{-9}) = 10 \log_{10} \frac{10^{-9}}{10^{-16}}$

4. $= 70$ decibels

5. (c) $\beta(10^{-6.5}) = 10 \log_{10} \frac{10^{-6.5}}{10^{-16}}$

6. $= 95$ decibels

7. (d) $\beta(10^{-4}) = 10 \log_{10} \frac{10^{-4}}{10^{-16}}$

8. $= 120$ decibels

69

The value of a tract of timber is

$V(t) = 100{,}000e^{0.8\sqrt{t}}$

where t is the time in years, with $t = 0$ corresponding to 1998. If money earns interest continuously at 10%, the present value of the timber at any time t is $A(t) = V(t)e^{-0.10t}$. Find the year in which the timber should be harvested to maximize the present value function.

1. $A(t) = V(t)e^{-0.10t}$
2. $\quad = 100{,}000e^{0.8\sqrt{t}}e^{-0.10t}$
3. $\quad = 100{,}000e^{0.8\sqrt{t}-0.10t}$
4. $\dfrac{dA}{dt} = 100{,}000\left(\dfrac{0.4}{\sqrt{t}} - 0.10\right)e^{0.8\sqrt{t}-0.10t}$
5. $\quad = 0$ when 16.
6. The timber should be harvested in the year 2014, $(1998 + 16)$.
7. **Note:** You could also use a graphing utility to graph $A(t)$ and find the maximum of $A(t)$. Use the viewing rectangle $0 \le x \le 30$ and $0 \le y \le 600{,}000$.

When an object is removed from a furnace and placed in an environment with a constant temperature of 80°F, its core temperature is 1500°F. One hour after it is removed, the core temperature is 1120°F. Find the core temperature 5 hours after the object is removed from the furnace.

1. Since $\frac{dy}{dt} = k(y - 80)$

2. $\int \frac{1}{y - 80}\, dy = \int k\, dt$

3. $\ln(y - 80) = kt + C.$

4. When $t = 0$, $y = 1500$.

5. Thus, $C = \ln 1420$.

6. When $t = 1$, $y = 1120$.

7. Thus,

   $k(1) + \ln 1420 = \ln(1120 - 80)$

8. $k = \ln 1040 - \ln 1420$

9. $= \ln \frac{104}{142}.$

10. Thus, $y = 1420e^{[\ln(104/142)]t} + 80$.

11. When $t = 5$, $y \approx 379.2°$.

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Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

In exponential growth, the rate of growth is constant.

1. False

2. If $y = Ce^{kt}$, $y' = Cke^{kt} \neq$ constant.

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Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

If prices are rising at a rate of 0.5% per month, then they are rising at a rate of 6% per year.

1. True

The table shows the population P (in millions) of the United States from 1960 to 2000. *(Source: U.S. Census Bureau)*

Year	1960	1970	1980	1990	2000
Population, P	181	205	228	250	282

(a) Use the 1960 and 1970 data to find an exponential model P_1 for the data. Let $t = 0$ represent 1960.

(b) Use a graphing utility to find an exponential model P_2 for the data. Let $t = 0$ represent 1960.

(c) Use a graphing utility to plot the data and graph both models in the same viewing window. Compare the actual data with the predictions. Which model better fits the data?

(d) Estimate when the population will be 320 million.

1. (a) $P_1 = Ce^{kt}$

2. $= 181e^{kt}$

3. $205 = 181e^{10k}$

4. $\Rightarrow k = \frac{1}{10}\ln\left(\frac{205}{181}\right)$

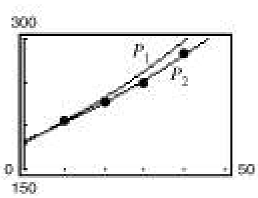
5. ≈ 0.01245

6. $P_1 \approx 181e^{0.01245t}$

7. $\approx 181(1.01253)^t$

8. (b) Using a graphing utility,
$P_2 \approx 182.3248(1.01091)^t$

9. (c)



10. The model P_2 fits the data better.

11. (d) Using the model P_2,

$$320 = 182.3248(1.01091)^t$$

12. $\frac{320}{182.3248} = (1.01091)^t$

13. $t = \frac{\ln(320/182.3248)}{\ln(1.01091)}$

14. ≈ 51.8 years, or 2011.