Exercises for Section 6.3

The symbol indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system.

Click on [S] to view the complete solution of the exercise.

Click on [M] to print an enlarged copy of the graph.

In Exercises 1–12, find the general solution of the differential equation.

[S] 1. $\frac{dy}{dx} = \frac{x}{y}$ 2. $\frac{dy}{dx} = \frac{x^2 + 2}{3y^2}$

[S] 3. $\frac{dr}{ds} = 0.05r$ 4. $\frac{dr}{ds} = 0.05s$

[S] 5. $(2 + x)y' = 3y$ 6. $xy' = y$

[S] 7. $yy' = \sin x$ 8. $yy' = 6\cos(\pi x)$

[S] 9. $\sqrt{1 - 4x^2}\, y' = x$ 10. $\sqrt{x^2 - 9}\, y' = 5x$

[S] 11. $y \ln x - xy' = 0$ 12. $4yy' - 3e^x = 0$

In Exercises 13–22, find the particular solution that satisfies the initial condition.

	Differential Equation	*Initial Condition*
[S]	13. $yy' - e^x = 0$	$y(0) = 4$
	14. $\sqrt{x} + \sqrt{y}\, y' = 0$	$y(1) = 4$
[S]	15. $y(x + 1) + y' = 0$	$y(-2) = 1$
	16. $2xy' - \ln x^2 = 0$	$y(1) = 2$
[S]	17. $y(1 + x^2)y' - x(1 + y^2) = 0$	$y(0) = \sqrt{3}$
	18. $y\sqrt{1 - x^2}\, y' - x\sqrt{1 - y^2} = 0$	$y(0) = 1$
[S]	19. $\frac{du}{dv} = uv \sin v^2$	$u(0) = 1$
	20. $\frac{dr}{ds} = e^{r - 2s}$	$r(0) = 0$
[S]	21. $dP - kP\, dt = 0$	$P(0) = P_0$
	22. $dT + k(T - 70)\, dt = 0$	$T(0) = 140$

In Exercises 23 and 24, find an equation of the graph that passes through the point and has the given slope.

[S] 23. $(1, 1)$, $y' = -\frac{9x}{16y}$

24. $(8, 2)$, $y' = \frac{2y}{3x}$

In Exercises 25 and 26, find all functions *f* having the indicated property.

[S] 25. The tangent to the graph of f at the point (x, y) intersects the x-axis at $(x + 2, 0)$.

26. All tangents to the graph of f pass through the origin.

In Exercises 27–34, determine whether the function is homogeneous, and if it is, determine its degree.

[S] 27. $f(x, y) = x^3 - 4xy^2 + y^3$ 28. $f(x, y) = x^3 + 3x^2y^2 - 2y^2$

[S] 31. $f(x, y) = 2 \ln xy$ 32. $f(x, y) =$

[S] 33. $f(x, y) = 2 \ln \frac{x}{y}$ 34. $f(x, y) =$

In Exercises 35–40, solve the homogeneous di

[S] 35. $y' = \frac{x + y}{2x}$ 36. $y' = \frac{x^3}{}$

[S] 37. $y' = \frac{x - y}{x + y}$ 38. $y' = \frac{x^2}{}$

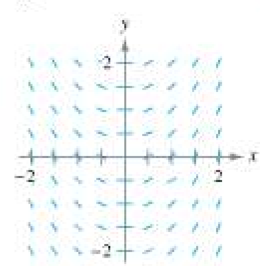
[S] 39. $y' = \frac{xy}{x^2 - y^2}$ 40. $y' = \frac{2x}{}$

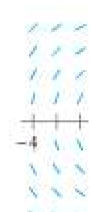
In Exercises 41–44, find the particular soluti initial condition.

	Differential Equation	*Initial*
[S]	41. $x\, dy - (2xe^{-y/x} + y)\, dx = 0$	$y(1) =$
	42. $-y^2\, dx + x(x + y)\, dy = 0$	$y(1) =$
[S]	43. $\left(x \sec \frac{y}{x} + y\right) dx - x\, dy = 0$	$y(1) =$
	44. $(2x^2 + y^2)\, dx + xy\, dy = 0$	$y(1) =$

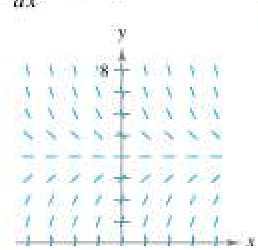
Slope Fields **In Exercises 45–48, sketch a f differential equation on the slope field and th solution analytically. To print an enlarged select the MathGraph button.**

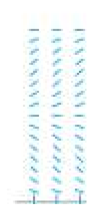
[M] [S] 45. $\frac{dy}{dx} = x$ [M] 46. $\frac{dy}{dx} = -$





[M] [S] 47. $\frac{dy}{dx} = 4 - y$ [M] 48. $\frac{dy}{dx} = 0.$





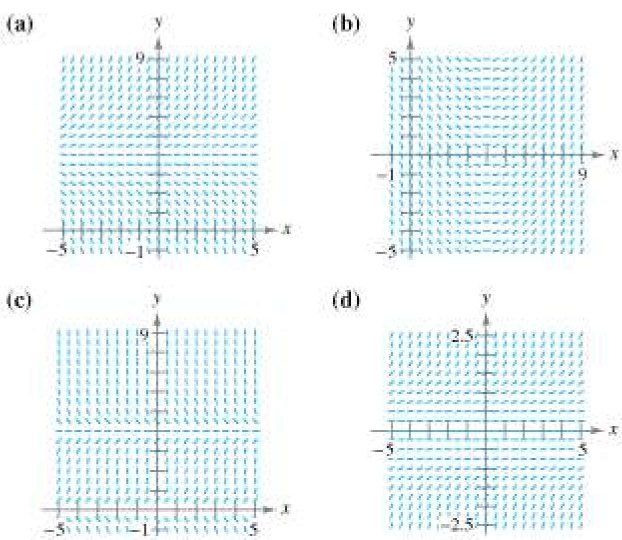
Euler's Method **In Exercises 49–52, (a) use Euler's Method with a step size of $h = 0.1$ to approximate the particular solution of the initial value problem at the given x-value, (b) find the exact solution of the differential equation analytically, and (c) compare the solutions at the given x-value.**

Differential Equation	*Initial Condition*	*x-value*
49. $\frac{dy}{dx} = -6xy$	$(0, 5)$	$x = 1$
50. $\frac{dy}{dx} + 6xy^2 = 0$	$(0, 3)$	$x = 1$
51. $\frac{dy}{dx} = \frac{2x + 12}{3y^2 - 4}$	$(1, 2)$	$x = 2$
52. $\frac{dy}{dx} = 2x(1 + y^2)$	$(1, 0)$	$x = 1.5$

53. ***Radioactive Decay*** The rate of decomposition of radioactive radium is proportional to the amount present at any time. The half-life of radioactive radium is 1599 years. What percent of a present amount will remain after 25 years?

54. ***Chemical Reaction*** In a chemical reaction, a certain compound changes into another compound at a rate proportional to the unchanged amount. If initially there are 20 grams of the original compound, and there is 16 grams after 1 hour, when will 75 percent of the compound be changed?

Slope Fields **In Exercises 55–58, (a) write a differential equation for the statement, (b) match the differential equation with a possible slope field, and (c) verify your result by using a graphing utility to graph a slope field for the differential equation. [The slope fields are labeled (a), (b), (c), and (d).] To print an enlarged copy of the graph, select the MathGraph button.**



55. The rate of change of y with respect to x is proportional to the difference between y and 4.

56. The rate of change of y with respect to x is proportional to the difference between x and 4.

57. The rate of change of y with respect to x is product of y and the difference between y

58. The rate of change of y with respect to x is

59. ***Weight Gain*** A calf that weighs 60 po weight at the rate

$$\frac{dw}{dt} = k(1200 - w)$$

where w is weight in pounds and t is time differential equation.

(a) Use a computer algebra system to so equation for $k = 0.8, 0.9$, and 1. Graph

(b) If the animal is sold when its weight r find the time of sale for each of the mo

(c) What is the maximum weight of the an models?

60. ***Weight Gain*** A calf that weighs w_0 po weight at the rate

$$\frac{dw}{dt} = 1200 - w$$

where w is weight in pounds and t is time differential equation.

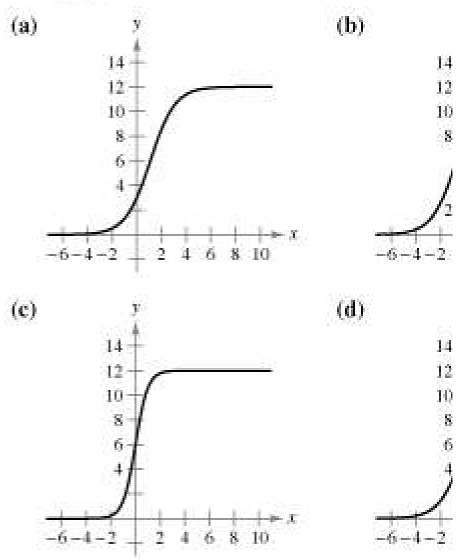
In Exercises 61–66, find the orthogonal family. Use a graphing utility to graph severa family.

61. $x^2 + y^2 = C$ **62.** $x^2 - 2y$

63. $x^2 = Cy$ **64.** $y^2 = 2C$

65. $y^2 = Cx^3$ **66.** $y = Ce^{x}$

In Exercises 67–70, match the logistic equat [The graphs are labeled (a), (b), (c), and (d).



S 67. $y = \dfrac{12}{1 + e^{-x}}$

68. $y = \dfrac{12}{1 + 3e^{-x}}$

S 69. $y = \dfrac{12}{1 + \frac{1}{2}e^{-x}}$

70. $y = \dfrac{12}{1 + e^{-2x}}$

In Exercises 71 and 72, the logistic equation models the growth of a population. Use the equation to (a) find the value of k, (b) find the carrying capacity, (c) find the initial population, (d) determine when the population will reach 50% of its carrying capacity, and (e) write a logistic differential equation that has the solution $P(t)$.

S 71. $P(t) = \dfrac{1500}{1 + 24e^{-0.75t}}$

72. $P(t) = \dfrac{5000}{1 + 39e^{-0.2t}}$

In Exercises 73 and 74, the logistic differential equation models the growth rate of a population. Use the equation to (a) find the value of k, (b) find the carrying capacity, (c) use a computer algebra system to graph a slope field, and (d) determine the value of P at which the population growth rate is the greatest.

S 73. $\dfrac{dP}{dt} = 3P\left(1 - \dfrac{P}{100}\right)$

74. $\dfrac{dP}{dt} = 0.1P - 0.0004P^2$

In Exercises 75–78, find the logistic equation that satisfies the initial condition.

Logistic Differential Equation	*Initial Condition*
S 75. $\dfrac{dy}{dt} = y\left(1 - \dfrac{y}{40}\right)$	$(0, 8)$
76. $\dfrac{dy}{dt} = 1.2y\left(1 - \dfrac{y}{8}\right)$	$(0, 5)$
S 77. $\dfrac{dy}{dt} = \dfrac{4y}{5} - \dfrac{y^2}{150}$	$(0, 8)$
78. $\dfrac{dy}{dt} = \dfrac{3y}{20} - \dfrac{y^2}{1600}$	$(0, 15)$

S 79. ***Endangered Species*** A conservation organization releases 25 Florida panthers into a game preserve. After 2 years, there are 39 panthers in the preserve. The Florida preserve has a carrying capacity of 200 panthers.

(a) Write a logistic equation that models the population of the panther population of the preserve.

(b) Find the population of the herd after 5 years.

(c) When will the herd's population reach 100?

(d) Write a logistic differential equation that models the growth rate of the panther population. Then repeat part (b) using Euler's Method with a step size of $h = 1$. Compare the approximation with the exact answers.

(e) At what time is the panther population growing most rapidly? Explain.

80. ***Bacteria Growth*** At time $t = 0$, a bacterial culture weighs 1 gram. Two hours later, the culture weighs 2 grams. The maximum weight of the culture is 10 grams.

(a) Write a logistic equation that models bacterial culture.

(b) Find the culture's weight after 5 hours.

(c) When will the culture's weight reach 8

(d) Write a logistic differential equatio growth rate of the culture's weight. T using Euler's Method with a step size the approximation with the exact answ

(e) At what time is the culture's weig rapidly? Explain.

Writing About Concepts

S 81. In your own words, describe how to re differential equations that can be solved variables.

82. State the test for determining if a diffe homogeneous. Give an example.

S 83. In your own words, describe the relation families of curves that are mutually orth

84. ***Sailing*** Ignoring resistance, a sailboat accelerates (dv/dt) at a rate proportiona between the velocities of the wind and the

(a) The wind is blowing at 20 knots, and aft is moving at 5 knots. Write the velocit time t.

(b) Use the result of part (a) to write the the boat as a function of time.

True or False? **In Exercises 85–88, deter statement is true or false. If it is false, expl example that shows it is false.**

S 85. The function $y = 0$ is always a solutio equation that can be solved by separation o

86. The differential equation $y' = xy - 2y +$ in separated variables form.

S 87. The function $f(x, y) = x^2 + xy + 2$ is hon

88. The families $x^2 + y^2 = 2Cy$ and $x^2 + y^2$ orthogonal.

S 89. Show that if $y = \dfrac{1}{1 + be^{-kt}}$, then $\dfrac{dy}{dt} = ky($

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90. A not uncommon calculus mistake is to bel rule for derivatives says that $(fg)' =$ determine, with proof, whether there exis (a, b) and a nonzero function g defined on wrong product rule is true for x in (a, b).