1

Find the general solution of the differential equation.

$$\frac{dy}{dx} = \frac{x}{y}$$

1. $\int y\,dy = \int x\,dx$
2. $\frac{y^2}{2} = \frac{x^2}{2} + C_1$
3. $y^2 - x^2 = C$

3

Find the general solution of the differential equation.

$$\frac{dr}{ds} = 0.05r$$

1. $\int \frac{dr}{r} = \int 0.05\,ds$
2. $\ln|r| = 0.05s + C_1$
3. $r = e^{0.05s + C_1}$
4. $= Ce^{0.05s}$

5.

Find the general solution of the differential equation.

$$(2 + x)y' = 3y$$

1. $\int \frac{dy}{y} = \int \frac{3}{2 + x}\,dx$
2. $\ln|y| = 3\ln|2 + x| + \ln C$
3. $= \ln|C(2 + x)^3|$
4. $y = C(x + 2)^3$

7

Find the general solution of the differential equation.

$yy' = \sin x$

1. $\int y\,dy = \int \sin x\,dx$
2. $\frac{y^2}{2} = -\cos x + C_1$
3. $y^2 = -2\cos x + C$

9

Find the general solution of the differential equation.

$\sqrt{1 - 4x^2}\,y' = x$

1. $dy = \frac{x}{\sqrt{1 - 4x^2}}\,dx$
2. $\int dy = \int \frac{x}{\sqrt{1 - 4x^2}}\,dx$
3. $= -\frac{1}{8}\int (1 - 4x^2)^{-1/2}(-8x\,dx)$
4. $y = -\frac{1}{4}(1 - 4x^2)^{1/2} + C$

11

Find the general solution of the differential equation.

$y \ln x - xy' = 0$

1. $\int \frac{dy}{y} = \int \frac{\ln x}{x}\,dx$
2. $\left(u = \ln x,\ du = \frac{dx}{x}\right)$
3. $\ln|y| = \frac{1}{2}(\ln x)^2 + C_1$
4. $y = e^{(1/2)(\ln x)^2 + C_1}$
5. $= Ce^{(\ln x)^2/2}$

13

Find the particular solution that satisfies the initial condition.

Differential Equation	*Initial Condition*
$yy' - e^x = 0$	$y(0) = 4$

1. $\int y\,dy = \int e^x\,dx$
2. $\frac{y^2}{2} = e^x + C_1$
3. $y^2 = 2e^x + C$
4. Initial condition: $y(0) = 4,\ 16 = 2 + C,\ C = 14$
5. Particular solution: $y^2 = 2e^x + 14$

15

Find the particular solution that satisfies the initial condition.

Differential Equation	*Initial Condition*
$y(x + 1) + y' = 0$	$y(-2) = 1$

1. $\int \frac{dy}{y} = -\int (x + 1)\,dx$
2. $\ln|y| = -\frac{(x + 1)^2}{2} + C_1$
3. $y = Ce^{-(x+1)^2/2}$
4. Initial condition: $y(-2) = 1,\ 1 = Ce^{-1/2},\ C = e^{1/2}$
5. Particular solution: $y = e^{[1-(x+1)^2]/2}$
6. $= e^{-(x^2+2x)/2}$

17

Find the particular solution that satisfies the initial condition.

Differential Equation	*Initial Condition*
$y(1+x^2)y' - x(1+y^2) = 0$	$y(0) = \sqrt{3}$

1. $\dfrac{y}{1+y^2}\,dy = \dfrac{x}{1+x^2}\,dx$
2. $\dfrac{1}{2}\ln(1+y^2) = \dfrac{1}{2}\ln(1+x^2) + C_1$
3. $\ln(1+y^2) = \ln(1+x^2) + \ln C$
4. $= \ln[C(1+x^2)]$
5. $1 + y^2 = C(1+x^2)$
6. $y(0) = \sqrt{3}$: $1 + 3 = C$
7. $\Longrightarrow C = 4$
8. $1 + y^2 = 4(1+x^2)$
9. $y^2 = 3 + 4x^2$

19

Find the particular solution that satisfies the initial condition.

Differential Equation	*Initial Condition*
$\dfrac{du}{dv} = uv \sin v^2$	$u(0) = 1$

1. $\displaystyle\int \frac{du}{u} = \int v \sin v^2\,dv$
2. $\ln|u| = -\dfrac{1}{2}\cos v^2 + C_1$
3. $u = Ce^{-(\cos v^2)/2}$
4. Initial condition: $u(0) = 1,\ C = \dfrac{1}{e^{-1/2}}$
5. $= e^{1/2}$
6. Particular solution: $u = e^{(1-\cos v^2)/2}$

21

Find the particular solution that satisfies the initial condition.

Differential Equation	*Initial Condition*
$dP - kP\,dt = 0$	$P(0) = P_0$

1. $\displaystyle\int \frac{dP}{P} = k\int dt$
2. $\ln|P| = kt + C_1$
3. $\quad P = Ce^{kt}$
4. Initial condition: $P(0) = P_0,\ P_0 = Ce^0$
5. $\qquad\qquad\qquad\qquad = C$
6. Particular solution: $P = P_0e^{kt}$

23

Find an equation of the graph that passes through the point and has the given slope.

$(1, 1),\ y' = -\dfrac{9x}{16y}$

1. $\displaystyle\int 16y\,dy = -\int 9x\,dx$
2. $\quad 8y^2 = \dfrac{-9}{2}x^2 + C$
3. Initial condition: $y(1) = 1,\ 8 = -\dfrac{9}{2} + C,\ C = \dfrac{25}{2}$
4. Particular solution: $\quad 8y^2 = \dfrac{-9}{2}x^2 + \dfrac{25}{2}$
5. $\quad 16y^2 + 9x^2 = 25$

25

Find all functions f having the indicated property.

The tangent to the graph of f at the point (x, y) intersects the x-axis at $(x + 2, 0)$.

1. $m = \frac{dy}{dx}$

2. $= \frac{0 - y}{(x + 2) - x}$

3. $= -\frac{y}{2}$

4. $\int \frac{dy}{y} = \int -\frac{1}{2}\, dx$

5. $\ln|y| = -\frac{1}{2}x + C_1$

6. $y = Ce^{-x/2}$

27

Determine whether the function is homogeneous, and if it is, determine its degree.

$f(x, y) = x^3 - 4xy^2 + y^3$

1. $f(tx, ty) = t^3x^3 - 4txt^2y^2 + t^3y^3$

2. $= t^3(x^3 - 4xy^2 + y^3)$

3. Homogeneous of degree 3

29

Determine whether the function is homogeneous, and if it is, determine its degree.

$f(x, y) = \frac{x^2y^2}{\sqrt{x^2 + y^2}}$

1. $f(tx, ty) = \frac{t^4x^2y^2}{\sqrt{t^2x^2 + t^2y^2}}$

2. $= t^3\frac{x^2y^2}{\sqrt{x^2 + y^2}}$

3. Homogeneous of degree 3

31

Determine whether the function is homogeneous, and if it is, determine its degree.

$f(x, y) = 2 \ln xy$

1. $f(tx, ty) = 2 \ln[txty]$
2. $= 2 \ln[t^2 xy]$
3. $= 2(\ln t^2 + \ln xy)$
4. Not homogeneous

33

Determine whether the function is homogeneous, and if it is, determine its degree.

$f(x, y) = 2 \ln \frac{x}{y}$

1. $f(tx, ty) = 2 \ln \frac{tx}{ty}$
2. $= 2 \ln \frac{x}{y}$
3. Homogeneous degree 0

35

Solve the homogeneous differential equation.

$$y' = \frac{x + y}{2x}$$

1. $$y = vx$$

2. $$v + x\frac{dv}{dx} = \frac{x + vx}{2x}$$

3. $$x\frac{dv}{dx} = \frac{1 + v}{2} - v$$

4. $$= \frac{1 - v}{2}$$

5. $$2\int \frac{dv}{1 - v} = \int \frac{dx}{x}$$

6. $$-\ln(1 - v)^2 = \ln|x| + \ln C$$

7. $$= \ln|Cx|$$

8. $$\frac{1}{(1 - v)^2} = |Cx|$$

9. $$\frac{1}{[1 - (y/x)]^2} = |Cx|$$

10. $$\frac{x^2}{(x - y)^2} = |Cx|$$

11. $$|x| = C(x - y)^2$$

Solve the homogeneous differential equation.

$$y' = \frac{x - y}{x + y}$$

1. $$y = vx$$

2. $$v + x\frac{dv}{dx} = \frac{x - xv}{x + xv}$$

3. $$v\,dx + x\,dv = \frac{1 - v}{1 + v}\,dx$$

4. $$x\,dv = \left(\frac{1 - v}{1 + v} - v\right)dx$$

5. $$= \frac{1 - 2v - v^2}{1 + v}\,dx$$

6. $$\int \frac{v + 1}{v^2 + 2v - 1}\,dv = -\int \frac{dx}{x}$$

7. $$\frac{1}{2}\ln|v^2 + 2v - 1| = -\ln|x| + \ln C_1$$

8. $$= \ln\left|\frac{C_1}{x}\right|$$

9. $$|v^2 + 2v - 1| = \frac{C}{x^2}$$

10. $$\left|\frac{y^2}{x^2} + 2\frac{y}{x} - 1\right| = \frac{C}{x^2}$$

11. $$|y^2 + 2xy - x^2| = C$$

Solve the homogeneous differential equation.

$$y' = \frac{xy}{x^2 - y^2}$$

1. $$y = vx$$

2. $$v + x\frac{dv}{dx} = \frac{x^2 v}{x^2 - x^2 v^2}$$

3. $$v\,dx + x\,dv = \frac{v}{1 - v^2}\,dx$$

4. $$x\,dv = \left(\frac{v}{1 - v^2} - v\right)dx$$

5. $$= \left(\frac{v^3}{1 - v^2}\right)dx$$

6. $$\int \frac{1 - v^2}{v^3}\,dv = \int \frac{dx}{x}$$

7. $$-\frac{1}{2v^2} - \ln|v| = \ln|x| + \ln C_1$$

8. $$= \ln|C_1 x|$$

9. $$\frac{-1}{2v^2} = \ln|C_1 x v|$$

10. $$\frac{-x^2}{2y^2} = \ln|C_1 y|$$

11. $$y = Ce^{-x^2/2y^2}$$

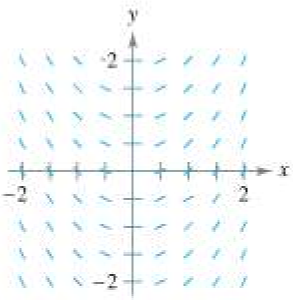
Find the particular solution that satisfies the initial condition.

Differential Equation	*Initial Condition*
$x\,dy - (2xe^{-y/x} + y)\,dx = 0$	$y(1) = 0$

1. $y = vx$
2. $x(v\,dx + x\,dv) - (2xe^{-v} + vx)\,dx = 0$
3. $\int e^{v}\,dv = \int \frac{2}{x}\,dx$
4. $e^{v} = \ln C_1 x^2$
5. $e^{y/x} = \ln C_1 + \ln x^2$
6. $e^{y/x} = C + \ln x^2$
7. Initial condition: $y(1) = 0,\ 1 = C$
8. Particular solution: $e^{y/x} = 1 + \ln x^2$

Sketch a few solutions of the differential equation on the slope field and then find the general solution analytically.

$$\frac{dy}{dx} = x$$



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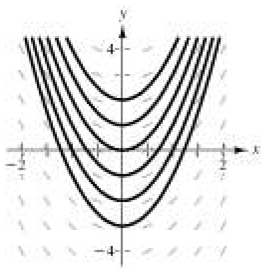
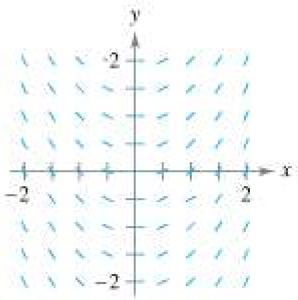


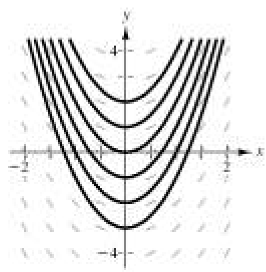
2. $y = \int x\,dx$

3. $= \frac{1}{2}x^2 + C$

Sketch a few solutions of the differential equation on the slope field and then find the general solution analytically.

$$\frac{dy}{dx} = x$$



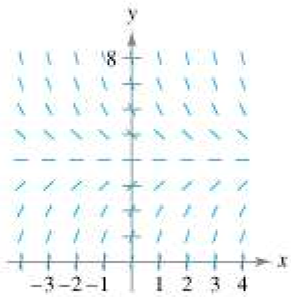
1. 

2. $y = \int x \, dx$

3. $\quad = \frac{1}{2}x^2 + C$

Sketch a few solutions of the differential equation on the slope field and then find the general solution analytically.

$$\frac{dy}{dx} = 4 - y$$



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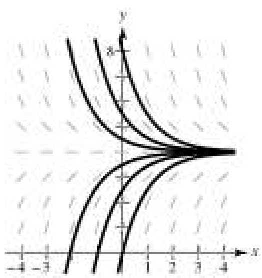


2. $$\int \frac{dy}{4 - y} = \int dx$$

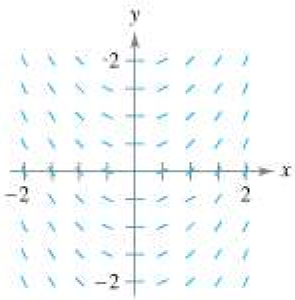
3. $\ln |4 - y| = -x + C_1$

4. $4 - y = e^{-x + C_1}$

5. $y = 4 + Ce^{-x}$

Sketch a few solutions of the differential equation on the slope field and then find the general solution analytically.

$$\frac{dy}{dx} = x$$



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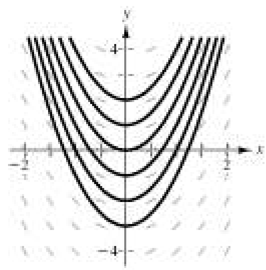


2. $y = \int x \, dx$

3. $\quad = \frac{1}{2}x^2 + C$

(a) Use Euler's Method with a step size of $h = 0.1$ to approximate the particular solution of the initial value problem at the given x-value, (b) find the exact solution of the differential equation analytically, and (c) compare the solutions at the given x-value.

Differential Equation	*Initial Condition*	*x-value*
$\dfrac{dy}{dx} = \dfrac{2x + 12}{3y^2 - 4}$	$(1, 2)$	$x = 2$

1. (a) Euler's Method gives $y(2) \approx 3.0318$.

2. (b) $$\int (3y^2 - 4)\, dy = \int (2x + 12)\, dx$$

3. $$y^3 - 4y = x^2 + 12x + C$$

4. $$y(1) = 2:\ 2^3 - 4(2) = 1 + 12 + C$$

5. $$\Rightarrow C = -13$$

6. $$y^3 - 4y = x^2 + 12x - 13$$

7. (c) For $x = 2$,

$$y^3 - 4y = 2^2 + 12(2) - 13$$

8. $$= 15$$

9. $$y^3 - 4y - 15 = 0$$

10. $$(y - 3)(y^2 + 3y + 5) = 0$$

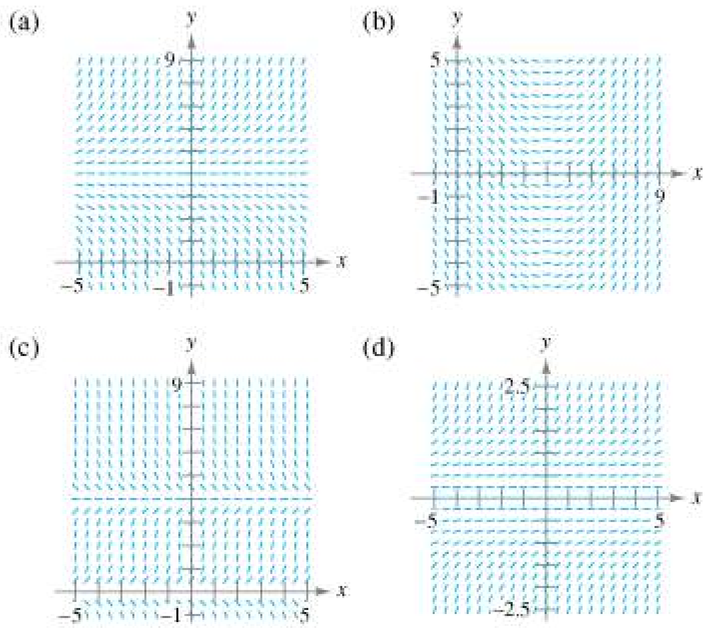
11. $$\Rightarrow y = 3$$

12. Error: $3.0318 - 3 = 0.0318$

The rate of decomposition of radioactive radium is proportional to the amount present at any time. The half-life of radioactive radium is 1599 years. What percent of a present amount will remain after 25 years?

1. $\frac{dy}{dt} = ky$
2. $y = Ce^{kt}$
3. $y(0) = y_0$
4. $= C$ initial amount
5. $\frac{y_0}{2} = y_0 e^{k(1599)}$
6. $k = \frac{1}{1599} \ln\left(\frac{1}{2}\right)$
7. $y = Ce^{[\ln(1/2)/1599]t}$
8. When $t = 25$, $y = 0.989C$ or 98.9%.

(a) Write a differential equation for the statement, (b) match the differential equation with a possible slope field, and (c) verify your result by using a graphing utility to graph a slope field for the differential equation. [The slope fields are labeled (a), (b), (c), and (d).]



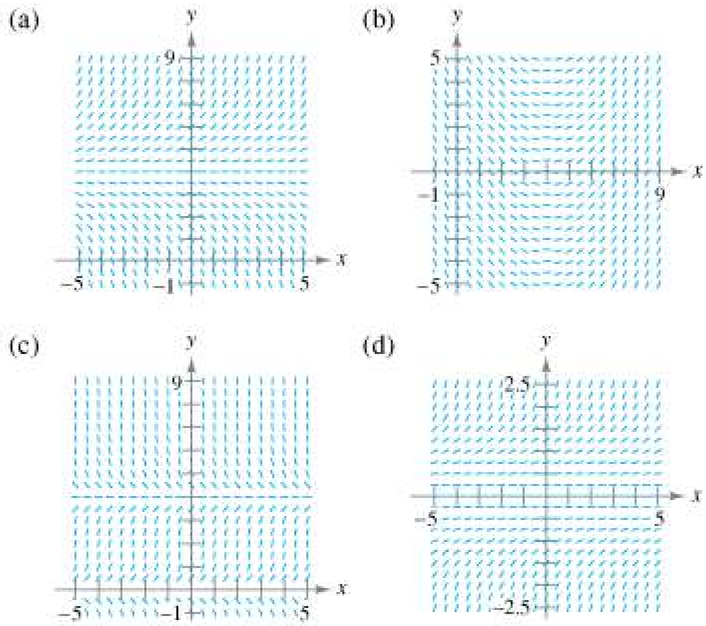
The rate of change of y with respect to x is proportional to the difference between y and 4.

1. $\dfrac{dy}{dx} = k(y - 4)$

2. The direction field satisfies $(dy/dx) = 0$ along $y = 4$; but not along $y = 0$.

3. Matches (a).

(a) Write a differential equation for the statement, (b) match the differential equation with a possible slope field, and (c) verify your result by using a graphing utility to graph a slope field for the differential equation. [The slope fields are labeled (a), (b), (c), and (d).]



The rate of change of y with respect to x is proportional to the product of y and the difference between y and 4.

1. $\dfrac{dy}{dx} = ky(y - 4)$

2. The direction field satisfies $(dy/dx) = 0$ along $y = 0$ and $y = 4$.

3. Matches (c).

A calf that weighs 60 pounds at birth gains weight at the rate

$$\frac{dw}{dt} = k(1200 - w)$$

where w is weight in pounds and t is time in years. Solve the differential equation.

(a) Use a computer algebra system to solve the differential equation for $k = 0.8, 0.9$, and 1. Graph the three solutions.

(b) If the animal is sold when its weight reaches 800 pounds, find the time of sale for each of the models in part (a).

(c) What is the maximum weight of the animal for each of the models?

1. $\displaystyle\int \frac{dw}{1200 - w} = \int k\,dt$

2. $\ln|1200 - w| = -kt + C_1$

3. $1200 - w = e^{-kt + C_1}$

4. $= Ce^{-kt}$

5. $w = 1200 - Ce^{-kt}$

6. $w(0) = 60$

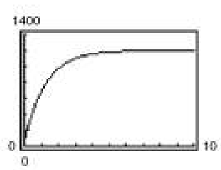
7. $= 1200 - C$

8. $\Rightarrow C = 1200 - 60$

9. $= 1140$

10. $w = 1200 - 1140e^{-kt}$

11. (a)



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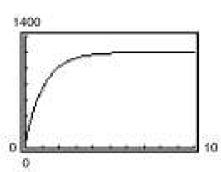

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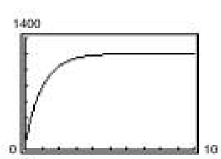


14. (b) $k = 0.8$: $t = 1.31$ years

15. $k = 0.9$: $t = 1.16$ years

16. $k = 1.0$: $t = 1.05$ years

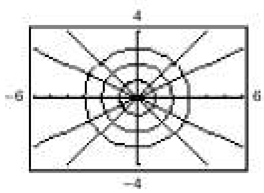
17. (c) Maximum weight: 1200 pounds

18. $\lim_{t \to \infty} w = 1200$

61

Find the orthogonal trajectories of the family. Use a graphing utility to graph several members of each family.

$x^2 + y^2 = C$

1. Given family (circles): $x^2 + y^2 = C$
2. $2x + 2yy' = 0$
3. $y' = -\dfrac{x}{y}$
4. Orthogonal trajectory (lines): $y' = \dfrac{y}{x}$
5. $\displaystyle\int \frac{dy}{y} = \int \frac{dx}{x}$
6. $\ln|y| = \ln|x| + \ln K$
7. $y = Kx$
8. 

63

Find the orthogonal trajectories of the family. Use a graphing utility to graph several members of each family.

$x^2 = Cy$

1. Given family (parabolas): $x^2 = Cy$

2. $$2x = Cy'$$

3. $$y' = \frac{2x}{C}$$

4. $$= \frac{2x}{x^2/y}$$

5. $$= \frac{2y}{x}$$

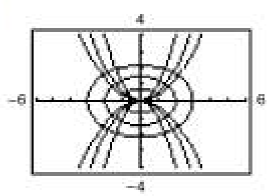
6. Orthogonal trajectory (ellipses): $y' = -\frac{x}{2y}$

7. $$2\int y\,dy = -\int x\,dx$$

8. $$y^2 = -\frac{x^2}{2} + K_1$$

9. $$x^2 + 2y^2 = K$$

10.



Find the orthogonal trajectories of the family. Use a graphing utility to graph several members of each family.

$y^2 = Cx^3$

1. Given family: $y^2 = Cx^3$

2. $2yy' = 3Cx^2$

3. $y' = \frac{3Cx^2}{2y}$

4. $= \frac{3x^2}{2y}\left(\frac{y^2}{x^3}\right)$

5. $= \frac{3y}{2x}$

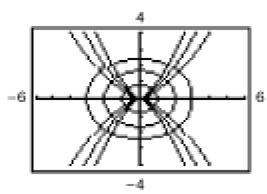
6. Orthogonal trajectory (ellipses): $y' = -\frac{2x}{3y}$

7. $3\int y\,dy = -2\int x\,dx$

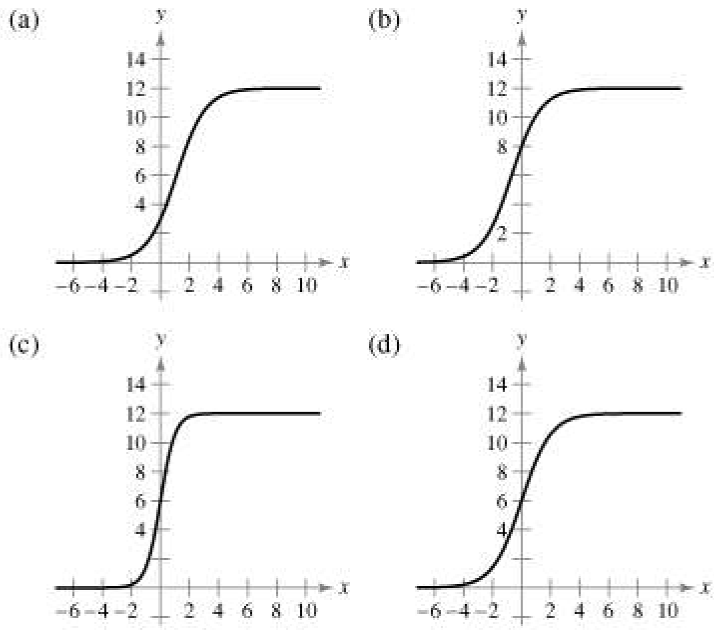
8. $\frac{3y^2}{2} = -x^2 + K_1$

9. $3y^2 + 2x^2 = K$

10.



Match the logistic equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]



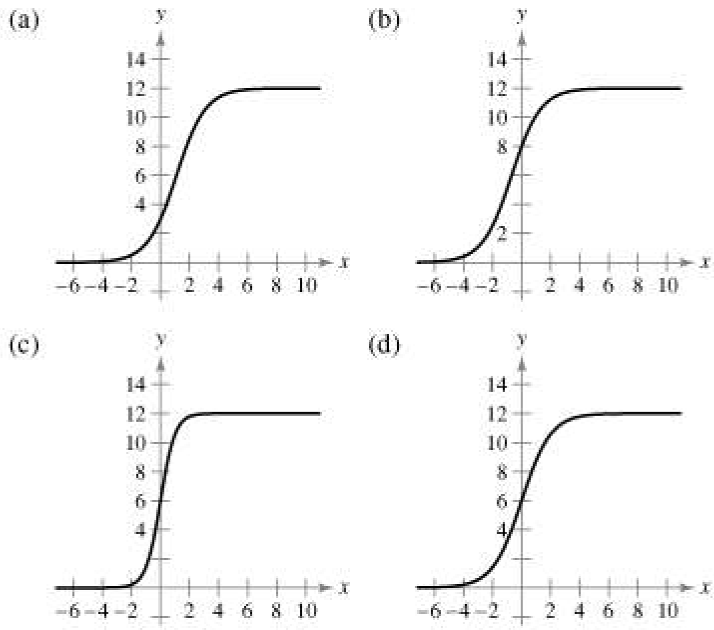
$$y = \frac{12}{1 + e^{-x}}$$

1. Since $y(0) = 6$, it matches (c) or (d).

2. Since (d) approaches its horizontal asymptote slower than (c)

3. it matches (d).

Match the logistic equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]



$$y = \frac{12}{1 + \frac{1}{2}e^{-x}}$$

1. Since $y(0) = \frac{12}{\left(\frac{3}{2}\right)}$

2. $= 8$

3. it matches (b).

The logistic equation models the growth of a population. Use the equation to (a) find the value of k, (b) find the carrying capacity, (c) find the initial population, (d) determine when the population will reach 50% of its carrying capacity, and (e) write a logistic differential equation that has the solution $P(t)$.

$$P(t) = \frac{1500}{1 + 24e^{-0.75t}}$$

1. (a) $k = 0.75$

2. (b) $L = 1500$

3. (c) $P(0) = \dfrac{1500}{1 + 24}$

4. $\quad = 60$

5. (d) $750 = \dfrac{1500}{1 + 24e^{-0.75t}}$

6. $1 + 24e^{-0.75t} = 2$

7. $e^{-0.75t} = \dfrac{1}{24}$

8. $-0.75t = \ln\left(\dfrac{1}{24}\right)$

9. $\quad = -\ln 24$

10. $t = \dfrac{\ln 24}{0.75}$

11. $\quad \approx 4.2374$

12. (e) $\dfrac{dP}{dt} = 0.75P\left(1 - \dfrac{P}{1500}\right)$

13. $P(0) = 60$

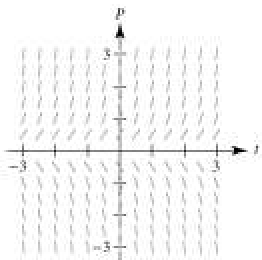
The logistic differential equation models the growth rate of a population. Use the equation to (a) find the value of k, (b) find the carrying capacity, (c) use a computer algebra system to graph a slope field, and (d) determine the value of P at which the population growth rate is the greatest.

$$\frac{dP}{dt} = 3P\left(1 - \frac{P}{100}\right)$$

1. (a) $k = 3$

2. (b) $L = 100$

3. (c)



4. (d) $\frac{d^2P}{dt^2} = 3P'\left(1 - \frac{P}{100}\right) + 3P\left(\frac{-P'}{100}\right)$

5. $= 3\left[3P\left(1 - \frac{P}{100}\right)\right]\left(1 - \frac{P}{100}\right) - \frac{3P}{100}\left[3P\left(1 - \frac{P}{100}\right)\right]$

6. $= 9P\left(1 - \frac{P}{100}\right)\left(1 - \frac{P}{100} - \frac{P}{100}\right)$

7. $= 9P\left(1 - \frac{P}{100}\right)\left(1 - \frac{2P}{100}\right)$

8. $\frac{d^2P}{dt^2} = 0$ for $P = 50$

9. and by the first Derivative Test, this is a maximum.

10. $\left(\text{Note: } P = 50 = \frac{L}{2} = \frac{100}{2}\right)$

Find the logistic equation that satisfies the initial condition.

Logistic Differential Equation	*Initial Condition*
$\frac{dy}{dt} = y\left(1 - \frac{y}{40}\right)$	(0, 8)

1. $k = 1$
2. $L = 40$
3. $y = \frac{L}{1 + be^{-kt}}$
4. $\quad = \frac{40}{1 + be^{-t}}$
5. $y(0) = 8$: $8 = \frac{40}{1 + b}$
6. $\qquad \Rightarrow b = 4$
7. Solution: $y = \frac{40}{1 + 4e^{-t}}$

Find the logistic equation that satisfies the initial condition.

Logistic Differential Equation	*Initial Condition*
$\frac{dy}{dt} = \frac{4y}{5} - \frac{y^2}{150}$	$(0, 8)$

1. $\frac{4y}{5} - \frac{y^2}{150} = \frac{4}{5}y\left(1 - \frac{y}{120}\right)$

2. $k = \frac{4}{5}$

3. $= 0.8$

4. $L = 120$

5. $y = \frac{L}{1 + be^{-kt}}$

6. $= \frac{120}{1 + be^{-0.8t}}$

7. $y(0) = 8$: $8 = \frac{120}{1 + b}$

8. $\Rightarrow b = 14$

9. Solution: $y = \frac{120}{1 + 14e^{-0.8t}}$

A conservation organization releases 25 Florida panthers into a game preserve. After 2 years, there are 39 panthers in the preserve. The Florida preserve has a carrying capacity of 200 panthers.

(a) Write a logistic equation that models the population of the panther population of the preserve.

(b) Find the population of the herd after 5 years.

(c) When will the herd's population reach 100?

(d) Write a logistic differential equation that models the growth rate of the panther population. Then repeat part (b) using Euler's Method with a step size of $h = 1$. Compare the approximation with the exact answers.

(e) At what time is the panther population growing most rapidly?

1. (a) $y = \dfrac{L}{1 + be^{-kt}}$

2. $L = 200$

3. $y(0) = 25$

4. $25 = \dfrac{200}{1 + b}$

5. $\Rightarrow b = 7$

6. $39 = \dfrac{200}{1 + 7e^{-k(2)}}$

7. $1 + 7e^{-2k} = \dfrac{200}{39}$

8. $e^{-2k} = \dfrac{23}{39}$

9. $k = -\dfrac{1}{2}\ln\left(\dfrac{23}{39}\right)$

10. $= \dfrac{1}{2}\ln\left(\dfrac{39}{23}\right)$

11. ≈ 0.2640

12. $y = \dfrac{200}{1 + 7e^{-0.2640t}}$

13. (b) For $t = 5$, $y \approx 70$ panthers.

14. (c) $100 = \dfrac{200}{1 + 7e^{-0.2640t}}$

15. $1 + 7e^{-0.264t} = 2$

16. $-0.264t = \ln\left(\dfrac{1}{7}\right)$

17. $t \approx 7.37$ years

18. (d) $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$

19. $= 0.264y\left(1 - \frac{y}{200}\right), \quad y(0) = 25$

20. Using Euler's Method, with $h = 1$, you obtain 65.6 panthers.

21. (e) y is increasing most rapidly where $y = \frac{200}{2} = 100$, corresponding to $t \approx 7.37$ years.

81

In your own words, describe how to recognize and solve differential equations that can be solved by separation of variables.

1. A differential equation can be solved by separation of variables if it can be written in the form

$$M(x) + N(y)\frac{dy}{dx} = 0.$$

2. To solve a separable equation, rewrite as $M(x)\,dx = -N(y)\,dy$ and integrate both sides.

83

In your own words, describe the relationship between two families of curves that are mutually orthogonal.

1. Two families of curves are mutually orthogonal if each curve in the first family intersects each curve in the second family at right angles.

85

Determine whether the statement is true or false.
If it is false, explain why or give an example that shows it is false.

The function $y = 0$ is always a solution of a differential equation that can be solved by separation of variables.

1. False
2. $\frac{dy}{dx} = \frac{x}{y}$ is separable
3. but $y = 0$ is not a solution.

87

Determine whether the statement is true or false.
If it is false, explain why or give an example that shows it is false.

The function $f(x, y) = x^2 + xy + 2$ is homogeneous.

1. False
2. $f(tx, ty) = t^2x^2 + t^2xy + 2$
3. $\qquad \neq t^2f(x, y)$

89

Show that if $y = \frac{1}{1 + be^{-kt}}$, then $\frac{dy}{dt} = ky(1 - y)$.

1. $y' = \frac{-1}{(1 + be^{-kt})^2}(-bke^{-kt})$
2. $\quad = \frac{k}{(1 + be^{-kt})} \cdot \frac{be^{-kt}}{(1 + be^{-kt})}$
3. $\quad = \frac{k}{(1 + be^{-kt})} \cdot \frac{1 + be^{-kt} - 1}{(1 + be^{-kt})}$
4. $\quad = \frac{k}{(1 + be^{-kt})} \cdot \left(1 - \frac{1}{1 + be^{-kt}}\right)$
5. $\quad = ky(1 - y)$