Exercises for Section 6.4

The symbol [graphing icon] indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system.

Click on [S] to view the complete solution of the exercise.

Click on [M] to print an enlarged copy of the graph.

In Exercises 1–4, determine whether the differential equation is linear. Explain your reasoning.

[S] **1.** $x^3y' + xy = e^x + 1$ **2.** $2xy - y' \ln x = y$

[S] **3.** $y' + y\cos x = xy^2$ **4.** $\dfrac{1 - y'}{y} = 3x$

In Exercises 5–14, solve the first-order linear differential equation.

[S] **5.** $\dfrac{dy}{dx} + \left(\dfrac{1}{x}\right)y = 3x + 4$ **6.** $\dfrac{dy}{dx} + \left(\dfrac{2}{x}\right)y = 3x + 2$

[S] **7.** $y' - y = 10$ **8.** $y' + 2xy = 4x$

[S] **9.** $(y + 1)\cos x\, dx - dy = 0$ **10.** $(y - 1)\sin x\, dx - dy = 0$

[S] **11.** $(x - 1)y' + y = x^2 - 1$ **12.** $y' + 3y = e^{3x}$

[S] **13.** $y' - 3x^2y = e^{x^3}$ **14.** $y' - y = \cos x$

[graphing icon] ***Slope Fields*** **In Exercises 15 and 16, (a) sketch an approximate solution of the differential equation satisfying the initial condition by hand on the slope field, (b) find the particular solution that satisfies the initial condition, and (c) use a graphing utility to graph the particular solution. Compare the graph with the hand-drawn graph of part (a). To print an enlarged copy of the graph, select the MathGraph button.**

Differential Equation	*Initial Condition*
[M] [S] **15.** $\dfrac{dy}{dx} = e^x - y$	$(0, 1)$
[M] **16.** $y' + \left(\dfrac{1}{x}\right)y = \sin x^2$	$(\sqrt{\pi}, 0)$

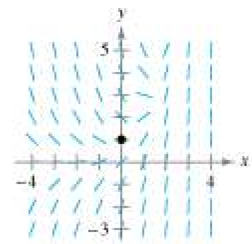

Figure for 15

Figure for 1

In Exercises 17–24, find the particular soluti tial equation that satisfies the boundary con

Differential Equation	*Bou*
[S] **17.** $y'\cos^2 x + y - 1 = 0$	$y(0)$
18. $x^3y' + 2y = e^{1/x^2}$	$y(1)$
[S] **19.** $y' + y\tan x = \sec x + \cos x$	$y(0)$
20. $y' + y\sec x = \sec x$	$y(0)$
[S] **21.** $y' + \left(\dfrac{1}{x}\right)y = 0$	$y(2)$
22. $y' + (2x - 1)y = 0$	$y(1)$
[S] **23.** $x\,dy = (x + y + 2)\,dx$	$y(1)$
24. $2xy' - y = x^3 - x$	$y(4)$

In Exercises 25–30, solve the Bernoulli differential equation.

25. $y' + 3x^2y = x^2y^3$

26. $y' + xy = xy^{-1}$

27. $y' + \left(\frac{1}{x}\right)y = xy^2$

28. $y' + \left(\frac{1}{x}\right)y = x\sqrt{y}$

29. $y' - y = e^x\sqrt[3]{y}$

30. $yy' - 2y^2 = e^x$

Slope Fields **In Exercises 31–34, (a) use a graphing utility to graph the slope field for the differential equation, (b) find the particular solutions of the differential equation passing through the given points, and (c) use a graphing utility to graph the particular solutions on the slope field.**

	Differential Equation	*Points*
31.	$\frac{dy}{dx} - \frac{1}{x}y = x^2$	$(-2, 4)$, $(2, 8)$
32.	$\frac{dy}{dx} + 4x^3y = x^3$	$\left(0, \frac{7}{2}\right)$, $\left(0, -\frac{1}{2}\right)$
33.	$\frac{dy}{dx} + (\cot x)y = 2$	$(1, 1)$, $(3, -1)$
34.	$\frac{dy}{dx} + 2xy = xy^2$	$(0, 3)$, $(0, 1)$

35. ***Population Growth*** When predicting population growth, demographers must consider birth and death rates as well as the net change caused by the difference between the rates of immigration and emigration. Let P be the population at time t and let N be the net increase per unit time resulting from the difference between immigration and emigration. So, the rate of growth of the population is given by

$$\frac{dP}{dt} = kP + N, \qquad N \text{ is constant.}$$

Solve this differential equation to find P as a function of time if at time $t = 0$ the size of the population is P_0.

36. ***Investment Growth*** A large corporation starts at time $t = 0$ to invest part of its receipts continuously at a rate of P dollars per year in a fund for future corporate expansion. Assume that the fund earns r percent interest per year compounded continuously. So, the rate of growth of the amount A in the fund is given by

$$\frac{dA}{dt} = rA + P$$

where $A = 0$ when $t = 0$. Solve this differential equation for A as a function of t.

Investment Growth **In Exercises 37 and 38, use the result of Exercise 36.**

37. Find A for the following.

(a) $P = \$100{,}000$, $r = 6\%$, and $t = 5$ years

(b) $P = \$250{,}000$, $r = 5\%$, and $t = 10$ years

38. Find t if the corporation needs \$800,000 and it can invest \$75,000 per year in a fund earning 8% interest compounded continuously.

39. ***Intravenous Feeding*** Glucose is added i bloodstream at the rate of q units per mi removes glucose from the bloodstream at a the amount present. Assume that $Q(t)$ is th in the bloodstream at time t.

(a) Determine the differential equation de change of glucose in the bloodstream w

(b) Solve the differential equation from par when $t = 0$.

(c) Find the limit of $Q(t)$ as $t \to \infty$.

40. ***Learning Curve*** The management at a found that the maximum number of units a in a day is 40. The rate of increase in the produced with respect to time t in days by proportional to $40 - N$.

(a) Determine the differential equation de change of performance with respect to

(b) Solve the differential equation from pa

(c) Find the particular solution for a n produced 10 units on the first day at the on the twentieth day.

Mixture **In Exercises 41–46, consider a tank contains v_0 gallons of a solution of which, by is soluble concentrate. Another solution cont the concentrate per gallon is running into the r_1 gallons per minute. The solution in the stirred and is withdrawn at the rate of r_2 gal**

41. If Q is the amount of concentrate in the so show that

$$\frac{dQ}{dt} + \frac{r_2Q}{v_0 + (r_1 - r_2)t} = q_1r_1.$$

42. If Q is the amount of concentrate in the so write the differential equation for the rate respect to t if $r_1 = r_2 = r$.

43. A 200-gallon tank is full of a solution cont concentrate. Starting at time $t = 0$, distille to the tank at a rate of 10 gallons pe well-stirred solution is withdrawn at the sa

(a) Find the amount of concentrate Q i function of t.

(b) Find the time at which the amount o tank reaches 15 pounds.

(c) Find the quantity of the concentrate $t \to \infty$.

44. Repeat Exercise 43, assuming that the s tank contains 0.04 pound of concentrate pe

45. A 200-gallon tank is half full of distilled w a solution containing 0.5 pound of concentr the tank at the rate of 5 gallons per minute, mixture is withdrawn at the rate of 3 gallo

(a) At what time will the tank be full?

(b) At the time the tank is full, how many trate will it contain?

46. Repeat Exercise 45, assuming that the solution entering the tank contains 1 pound of concentrate per gallon.

Falling Object **In Exercises 47 and 48, consider an eight-pound object dropped from a height of 5000 feet, where the air resistance is proportional to the velocity.**

47. Write the velocity as a function of time if its velocity after 5 seconds is approximately -101 feet per second. What is the limiting value of the velocity function?

48. Use the result of Exercise 47 to write the position of the object as a function of time. Approximate the velocity of the object when it reaches ground level.

Electric Circuits **In Exercises 49 and 50, use the differential equation for electric circuits given by**

$$L\frac{dI}{dt} + RI = E.$$

In this equation, I is the current, R is the resistance, L is the inductance, and E is the electromotive force (voltage).

49. Solve the differential equation given a constant voltage E_0.

50. Use the result of Exercise 49 to find the equation for the current if $I(0) = 0$, $E_0 = 120$ volts, $R = 600$ ohms, and $L = 4$ henrys. When does the current reach 90% of its limiting value?

Writing About Concepts

51. Give the standard form of a first-order linear differential equation. What is its integrating factor?

52. Give the standard form of the Bernoulli equation. Describe how one reduces it to a linear equation.

In Exercises 53–56, match the differential
solution.

Differential Equation	*Solution*
53. $y' - 2x = 0$	(a) $y = Ce^{x^2}$
54. $y' - 2y = 0$	(b) $y = -\frac{1}{2} +$
55. $y' - 2xy = 0$	(c) $y = x^2 + ($
56. $y' - 2xy = x$	(d) $y = Ce^{2x}$

In Exercises 57–68, solve the first-order diffe
any appropriate method.

57. $\dfrac{dy}{dx} = \dfrac{e^{2x+y}}{e^{x-y}}$

58. $\dfrac{dy}{dx} =$

59. $y\cos x - \cos x + \dfrac{dy}{dx} = 0$

60. $y' =$

61. $(3y^2 + 4xy)\,dx + (2xy + x^2)\,dy = 0$

62. $(x + y)\,dx - x\,dy = 0$

63. $(2y - e^x)\,dx + x\,dy = 0$

64. $(y^2 + xy)\,dx - x^2\,dy = 0$

65. $(x^2y^4 - 1)\,dx + x^3y^3\,dy = 0$

66. $y\,dx + (3x + 4y)\,dy = 0$

67. $3(y - 4x^2)\,dx + x\,dy = 0$

68. $x\,dx + (y + e^y)(x^2 + 1)\,dy = 0$

True or False? **In Exercises 69 and 70, dete**
statement is true or false. If it is false, expl
example that shows it is false.

69. $y' + x\sqrt{y} = x^2$ is a first-order linear differ

70. $y' + xy = e^x y$ is a first-order linear differe