1.

Determine whether the differential equation is linear. Explain your reasoning.

$x^3y' + xy = e^x + 1$

1. $y' + \frac{1}{x^2}y = \frac{1}{x^3}(e^x + 1)$

2. Linear

3

Determine whether the differential equation is linear. Explain your reasoning.

$y' + y\cos x = xy^2$

1. Not linear, because of the xy^2-term.

5

Solve the first-order linear differential equation.

$\frac{dy}{dx} + \left(\frac{1}{x}\right)y = 3x + 4$

1. Integrating factor: $e^{\int (1/x)\,dx} = e^{\ln x}$

2. $= x$

3. $xy = \int x(3x + 4)\,dx$

4. $= x^3 + 2x^2 + C$

5. $y = x^2 + 2x + \frac{C}{x}$

7

Solve the first-order linear differential equation.

$y' - y = 10$

1. Integrating factor: $e^{\int -1\,dx} = e^{-x}$
2. $e^{-x}y' - e^{-x}y = 10e^{-x}$
3. $ye^{-x} = \int 10e^{-x}\,dx$
4. $= -10e^{-x} + C$
5. $y = -10 + Ce^{x}$

9

Solve the first-order linear differential equation.

$(y + 1)\cos x\,dx - dy = 0$

1. $y' = (y + 1)\cos x$
2. $= y\cos x + \cos x$
3. $y' - (\cos x)y = \cos x$
4. Integrating factor: $e^{\int -\cos x\,dx} = e^{-\sin x}$
5. $y'e^{-\sin x} - (\cos x)e^{-\sin x}y = (\cos x)e^{-\sin x}$
6. $ye^{-\sin x} = \int (\cos x)e^{-\sin x}\,dx$
7. $= -e^{-\sin x} + C$
8. $y = -1 + Ce^{\sin x}$

11

Solve the first-order linear differential equation.

$(x - 1)y' + y = x^2 - 1$

1. $y' + \left(\frac{1}{x-1}\right)y = x + 1$

2. Integrating factor: $e^{\int [1/(x-1)]\,dx} = e^{\ln|x-1|}$

3. $= x - 1$

4. $y(x - 1) = \int (x^2 - 1)\,dx$

5. $= \frac{1}{3}x^3 - x + C_1$

6. $y = \frac{x^3 - 3x + C}{3(x - 1)}$

13

Solve the first-order linear differential equation.

$y' - 3x^2 y = e^{x^3}$

1. Integrating factor: $e^{-\int 3x^2\,dx} = e^{-x^3}$

2. $ye^{-x^3} = \int e^{x^3} e^{-x^3}\,dx$

3. $= \int dx$

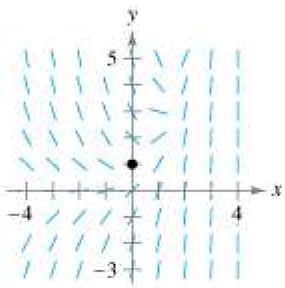
4. $= x + C$

5. $y = (x + C)e^{x^3}$

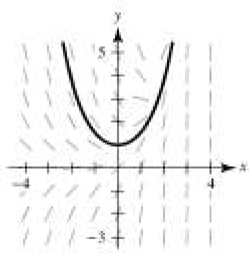
15

(a) Sketch an approximate solution of the differential equation satisfying the initial condition by hand on the slope field, (b) find the particular solution that satisfies the initial condition, and (c) use a graphing utility to graph the particular solution. Compare the graph with the hand-drawn graph of part (a).

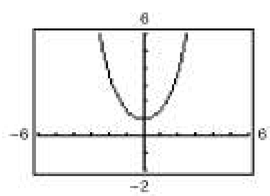
Differential Equation	*Initial Condition*
$\frac{dy}{dx} = e^x - y$	$(0, 1)$



1. (a),(c)



2.



3. (b) $\frac{dy}{dx} + y = e^x$

4. Integrating factor: $e^{\int dx} = e^x$

5. $e^x y' + e^x y = e^{2x}$

6. $(ye^x) = \int e^{2x}\, dx$

7. $ye^x = \frac{1}{2}e^{2x} + C$

8. $y(0) = 1$

9. $\Rightarrow 1 = \frac{1}{2} + C$

10. $\Rightarrow C = \frac{1}{2}$

11. $ye^x = \frac{1}{2}e^{2x} + \frac{1}{2}$

17

Find the particular solution of the differential equation that satisfies the boundary condition.

Differential Equation	*Boundary Condition*
$y' \cos^2 x + y - 1 = 0$	$y(0) = 5$

1. $y' + (\sec^2 x)y = \sec^2 x$
2. Integrating factor: $e^{\int \sec^2 x\, dx} = e^{\tan x}$
3. $ye^{\tan x} = \int \sec^2 x e^{\tan x}\, dx$
4. $= e^{\tan x} + C$
5. $y = 1 + Ce^{-\tan x}$
6. Initial condition: $y(0) = 5, C = 4$
7. Particular solution: $y = 1 + 4e^{-\tan x}$

19

Find the particular solution of the differential equation that satisfies the boundary condition.

Differential Equation	*Boundary Condition*
$y' + \left(\frac{1}{x}\right)y = 0$	$y(2) = 2$

1. Integrating factor: $e^{\int (1/x)\, dx} = e^{\ln|x|}$
2. $= x$
3. Separation of variables: $\frac{dy}{dx} = -\frac{y}{x}$
4. $\int \frac{1}{y}\, dy = \int -\frac{1}{x}\, dx$
5. $\ln y = -\ln x + \ln C$
6. $\ln xy = \ln C$
7. $xy = C$
8. Initial condition: $y(2) = 2, C = 4$
9. Particular solution: $xy = 4$

21

Find the particular solution of the differential equation that satisfies the boundary condition.

Differential Equation	*Boundary Condition*
$y' + \left(\frac{1}{x}\right)y = 0$	$y(2) = 2$

1. Integrating factor: $e^{\int (1/x)\,dx} = e^{\ln|x|}$
2. $= x$
3. Separation of variables: $\frac{dy}{dx} = -\frac{y}{x}$
4. $\int \frac{1}{y}\,dy = \int -\frac{1}{x}\,dx$
5. $\ln y = -\ln x + \ln C$
6. $\ln xy = \ln C$
7. $xy = C$
8. Initial condition: $y(2) = 2$, $C = 4$
9. Particular solution: $xy = 4$

Find the particular solution of the differential equation that satisfies the boundary condition.

Differential Equation	*Boundary Condition*
$x\,dy = (x + y + 2)\,dx$	$y(1) = 10$

1. $\dfrac{dy}{dx} = \dfrac{x + y + 2}{x}$
2. $= \dfrac{y}{x} + 1 + \dfrac{2}{x}$
3. $\dfrac{dy}{dx} - \dfrac{1}{x}y = 1 + \dfrac{2}{x}$, Linear
4. $u(x) = e^{\int -(1/x)\,dx}$
5. $= \dfrac{1}{x}$
6. $y = x\displaystyle\int \left(1 + \frac{2}{x}\right)\frac{1}{x}\,dx$
7. $= x\displaystyle\int \left(\frac{1}{x} + \frac{2}{x^2}\right)dx$
8. $= x\left[\ln|x| + \dfrac{-2}{x} + C\right]$
9. $= -2 + x\ln|x| + Cx$
10. $y(1) = 10$
11. $= -2 + C$
12. $\Rightarrow C = 12$
13. $y = -2 + x\ln|x| + 12x$

Solve the Bernoulli differential equation.

$y' + 3x^2y = x^2y^3$

1. $n = 3$
2. $Q = x^2$
3. $P = 3x^2$
4. $y^{-2}e^{\int(-2)3x^2\,dx} = \int(-2)x^2e^{\int(-2)3x^2\,dx}dx$
5. $y^{-2}e^{-2x^3} = -\int 2x^2e^{-2x^3}\,dx$
6. $y^{-2}e^{-2x^3} = \frac{1}{3}e^{-2x^3} + C$
7. $y^{-2} = \frac{1}{3} + Ce^{2x^3}$
8. $\frac{1}{y^2} = Ce^{2x^3} + \frac{1}{3}$

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Solve the Bernoulli differential equation.

$y' + \left(\frac{1}{x}\right)y = xy^2$

1. $n = 2$
2. $Q = x$
3. $P = x^{-1}$
4. $e^{\int-(1/x)\,dx} = e^{-\ln|x|}$
5. $= x^{-1}$
6. $y^{-1}x^{-1} = \int -x(x^{-1})\,dx$
7. $= -x + C$
8. $\frac{1}{y} = -x^2 + Cx$
9. $y = \frac{1}{Cx - x^2}$

29.

Solve the Bernoulli differential equation.

$y' - y = e^x \sqrt[3]{y}$

1. $n = \dfrac{1}{3}$

2. $Q = e^x$

3. $P = -1$

4. $e^{\int -(2/3)\,dx} = e^{-(2/3)x}$

5. $y^{2/3} e^{-(2/3)x} = \displaystyle\int \frac{2}{3} e^x e^{-(2/3)x}\,dx$

6. $\qquad = \displaystyle\int \frac{2}{3} e^{(1/3)x}\,dx$

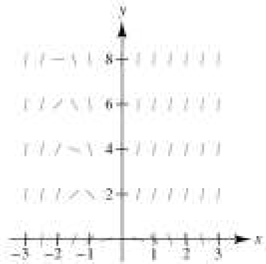
7. $y^{2/3} e^{-(2/3)x} = 2e^{(1/3)x} + C$

8. $\qquad y^{2/3} = 2e^x + Ce^{2x/3}$

(a) Use a graphing utility to graph the slope field for the differential equation, (b) find the particular solutions of the differential equation by passing through the given points, and (c) use a graphing utility to graph the particular solutions on the slope field.

Differential Equation	*Points*
$\frac{dy}{dx} - \frac{1}{x}y = x^2$	$(-2, 4), (2, 8)$

(a)



(b) Integrating factor: $e^{-\int 1/x\,dx} = e^{-\ln x}$

$$= \frac{1}{x}$$

$$\frac{1}{x}y' - \frac{1}{x^2}y = x$$

$$\left(\frac{1}{x}y\right) = \int x\,dx$$

$$= \frac{x^2}{x} + C$$

$$y = \frac{x^3}{2} + Cx$$

$$(-2, 4): 4 = \frac{-8}{2} - 2C$$

$$\Rightarrow C = -4$$

$$\Rightarrow y = \frac{x^3}{2} - 4x$$

$$= \frac{1}{2}x(x^2 - 8)$$

$$(2, 8): 8 = \frac{8}{2} + 2C$$

$$\Rightarrow C = 2$$

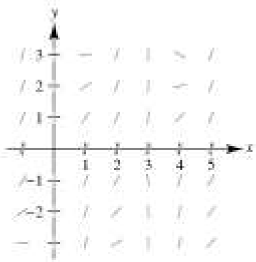
$$\Rightarrow y = \frac{x^3}{2} + 2x$$

$$= \frac{1}{2}x(x^2 + 4)$$

(a) Use a graphing utility to graph the slope field for the differential equation, (b) find the particular solutions of the differential equation by passing through the given points, and (c) use a graphing utility to graph the particular solutions on the slope field.

Differential Equation	*Points*
$\frac{dy}{dx} + (\cot x)y = 2$	$(1, 1), (3, -1)$

(a)



(b) Integrating factor: $e^{\int \cot x\,dx} = e^{\ln|\sin x|}$

$$= \sin x$$

$$y' \sin x + (\cos x)y = 2 \sin x$$

$$y \sin x = \int 2 \sin x\,dx$$

$$= -2 \cos x + C$$

$$y = -2 \cot x + C \csc x$$

$$(1, 1):\ 1 = -2 \cot 1 + C \csc 1$$

$$\Rightarrow C = \frac{1 + 2 \cot 1}{\csc 1}$$

$$= \sin 1 + 2 \cos 1$$

$$y = -2 \cot x + (\sin 1 + 2 \cos 1) \csc x$$

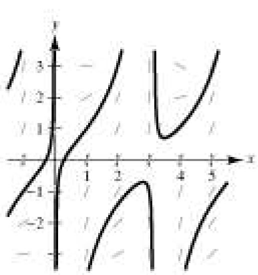
$$(3, -1):\ -1 = -2 \cot 3 + C \csc 3$$

$$\Rightarrow C = \frac{2 \cot 3 - 1}{\csc 3}$$

$$= 2 \cos 3 - \sin 3$$

$$y = -2 \cot x + (2 \cos 3 - \sin 3) \csc x$$

(c)



35

When predicting population growth, demographers must consider birth and death rates as well as the net change cause by the difference between the rates of immigration and emigration. Let P be the population at time t and let N be the net increase per unit time resulting from the difference between immigration and emigration. So, the rate of growth of the population is given by

$$\frac{dP}{dt} = kP + N, \quad N \text{ is constant.}$$

Solve this differential equation to find P as a function of time if at time $t = 0$ the size of the population is P_0.

1. $$\frac{dP}{kP + N} = dt$$
2. $$\int \frac{1}{kP + N}\,dP = \int dt$$
3. $$\frac{1}{k}\ln(kP + N) = t + C_1$$
4. $$\ln(kP + N) = kt + C_2$$
5. $$kP + N = e^{kt + C_2}$$
6. $$P = \frac{C_3 e^{kt} - N}{k}$$
7. $$P = Ce^{kt} - \frac{N}{k}$$
8. When $t = 0$: $P = P_0$
9. $$P_0 = C - \frac{N}{k}$$
10. $$\Rightarrow C = P_0 + \frac{N}{k}$$
11. $$P = \left(P_0 + \frac{N}{k}\right)e^{kt} - \frac{N}{k}$$

37

Find A for the following.

(a) $P = \$100{,}000$, $r = 6\%$, and $t = 5$ years

(b) $P = \$250{,}000$, $r = 5\%$, and $t = 10$ years

1. (a) $A = \frac{P}{r}(e^{rt} - 1)$

2. $A = \frac{100{,}000}{0.06}(e^{0.06(5)} - 1)$

3. $\approx 583{,}098.01$

4. (b) $A = \frac{250{,}000}{0.05}(e^{0.05(10)} - 1)$

5. $\approx 3{,}243{,}606.35$

Glucose is added intravenously to the bloodstream at the rate of q units per minute, and the body removes glucose from the bloodstream at a rate proportional to the amount present. Assume that $Q(t)$ is the amount of glucose in the bloodstream at time t.

(a) Determine the differential equation describing the rate of change of glucose in the bloodstream with respect to time.

(b) Solve the differential equation from part (a), letting $Q = Q_0$ when $t = 0$.

(c) Find the limit of $Q(t)$ as $t \to \infty$.

1. (a) $\dfrac{dQ}{dt} = q - kQ$, q constant

2. (b) $Q' + kQ = q$

3. Let $P(t) = k$, $Q(t) = q$,

4. then the integrating factor is $u(t) = e^{kt}$.

5. $Q = e^{-kt} \int q e^{kt}\, dt$

6. $= e^{-kt}\left(\dfrac{q}{k} e^{kt} + C\right)$

7. $= \dfrac{q}{k} + Ce^{-kt}$

8. When $t = 0$: $Q = Q_0$

9. $Q_0 = \dfrac{q}{k} + C$

10. $\Rightarrow C = Q_0 - \dfrac{q}{k}$

11. $Q = \dfrac{q}{k} + \left(Q_0 - \dfrac{q}{k}\right)e^{-kt}$

12. (c) $\lim\limits_{t \to \infty} Q = \dfrac{q}{k}$

Consider a tank that at time $t = 0$ contains v_0 gallons of a solution of which, by weight, q_0 pounds is soluble concentrate. Another solution containing q_1 pounds of the concentrate per gallon is running into the tank at the rate of r_1 gallons per minute. The solution in the tank is kept well stirred and is withdrawn at the rate of r_2 gallons per minute.

If Q is the amount of concentrate in the solution at any time t, show that

$$\frac{dQ}{dt} + \frac{r_2 Q}{v_0 + (r_1 - r_2)t} = q_1 r_1.$$

1. Let Q be the number of pounds of concentrate in the solution at any time t.
2. Since the number of gallons of solution in the tank at any time t is $v_0 + (r_1 - r_2)t$ and since the tank loses r_2 gallons of solution per minute, it must lose concentrate at the rate

   $$\left[\frac{Q}{v_0 + (r_1 - r_2)t}\right] r_2.$$
3. The solution gains concentrate at the rate $r_1 q_1$.
4. Therefore, the net rate of change is

   $$\frac{dQ}{dt} = q_1 r_1 - \left[\frac{Q}{v_0 + (r_1 - r_2)t}\right] r_2$$
5. or $\dfrac{dQ}{dt} + \dfrac{r_2 Q}{v_0 + (r_1 - r_2)t} = q_1 r_1.$

Consider a tank that at time $t = 0$ contains v_0 gallons of a solution of which, by weight, q_0 pounds is soluble concentrate. Another solution containing q_1 pounds of the concentrate per gallon is running into the tank at the rate of r_1 gallons per minute. The solution in the tank is kept well stirred and is withdrawn at the rate of r_2 gallons per minute.

A 200-gallon tank is full of a solution containing 25 pounds of concentrate. Starting at time $t = 0$, distilled water is admitted to the tank at a rate of 10 gallons per minute, and the well-stirred solution is withdrawn at the same rate.

(a) Find the amount of concentrate Q in the solution as a function of t.

(b) Find the time at which the amount of concentrate in the tank reaches 15 pounds.

(c) Find the quantity of the concentrate in the solution as $t \to \infty$.

1. (a) $Q' + \dfrac{r^2 Q}{v_0 + (r_1 - r_2)t} = q_1 r_1$

2. $Q(0) = q_0$

3. $q_0 = 25$

4. $q_1 = 0$

5. $v_0 = 200$

6. $r_1 = 10$

7. $r_2 = 10$

8. $Q' + \dfrac{1}{20}Q = 0$

9. $\displaystyle\int \frac{1}{Q}\,dQ = \int -\frac{1}{20}\,dt$

10. $\ln Q = -\dfrac{1}{20}t + \ln C_1$

11. $Q = Ce^{-(1/20)t}$

12. Initial condition: $Q(0) = 25$, $C = 25$

13. Particular solution: $Q = 25e^{-(1/20)t}$

14. (b) $15 = 25e^{-(1/20)t}$

15. $\ln\left(\dfrac{3}{5}\right) = -\dfrac{1}{20}t$

16. $t = -20\ln\left(\dfrac{3}{5}\right)$

17. ≈ 10.2 min

18. (c) $\displaystyle\lim_{t\to\infty} 25e^{-(1/20)t} = 0$

Consider a tank that at time $t = 0$ contains v_0 gallons of a solution of which, by weight, q_0 pounds is soluble concentrate. Another solution containing q_1 pounds of the concentrate per gallon is running into the tank at the rate of r_1 gallons per minute. The solution in the tank is kept well stirred and is withdrawn at the rate of r_2 gallons per minute.

A 200-gallon tank is half full of distilled water. At time $t = 0$, a solution containing 0.5 pound of concentrate per gallon enters the tank at the rate of 5 gallons per minute, and the well-stirred mixture is withdrawn at the rate of 3 gallons per minute.

(a) At what time will the tank be full?

(b) At the time the tank is full, how many pounds of concentrate will it contain?

1. (a) The volume of the solution in the tank is given by $v_0 + (r_1 - r_2)t$.

2. Therefore, $100 + (5 - 3)t = 200$ or $t = 50$ minutes.

3. (b) $Q' + \dfrac{r_2 Q}{v_0 + (r_1 - r_2)t} = q_1 r_1$

4. $Q(0) = q_0$

5. $q_0 = 0$

6. $q_1 = 0.5$

7. $v_0 = 100$

8. $r_1 = 5$

9. $r_2 = 3$

10. $Q' + \dfrac{3}{100 + 2t}Q = 2.5$

11. Integrating factor: $e^{\int [3/(100+2t)]\,dt} = (50 + t)^{3/2}$

12. $Q(50 + t)^{3/2} = \displaystyle\int 2.5(50 + t)^{3/2}\,dt$

13. $= (50 + t)^{5/2} + C$

14. $Q = (50 + t) + C(50 + t)^{-3/2}$

15. Initial condition: $Q(0) = 0$

16. $0 = 50 + C(50^{-3/2})$

17. $C = -50^{5/2}$

18. Particular solution: $Q = (50 + t) - 50^{-5/2}(50 + t)^{-3/2}$

19. $Q(50) = 100 - 50^{5/2}(100)^{-3/2}$

20. $= 100 - \dfrac{25}{\sqrt{2}}$

21. ≈ 82.32 lbs

47

Consider an eight-pound object dropped from a height of 5000 feet, where the air resistance is proportional to the velocity.

Write the velocity as a function of time if its velocity after 5 seconds is approximately -101 feet per second. What is the limiting value of the velocity function?

1. From Example 6,

$$\frac{dv}{dt} + \frac{kv}{m} = g$$

2. $v = \frac{mg}{k}(1 - e^{-kt/m})$.   Solution

3. $g = -32$

4. $mg = -8$

5. $v(5) = -101$

6. $m = \frac{-8}{g}$

7. $\quad = \frac{1}{4}$

8. implies that $-101 = \frac{-8}{k}(1 - e^{-5k/(1/4)})$.

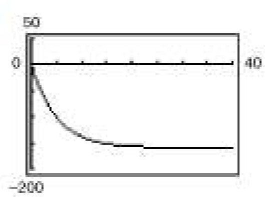
9. Using a graphing utility, $k \approx 0.050165$, and

10. $v = -159.47(1 - e^{-0.2007t})$.

11. As $t \to \infty$, $v \to -159.47$ ft/sec.

12. The graph of v is shown below.

13.



49

Use the differential equation for electric circuits given by

$$L\frac{dI}{dt} + RI = E.$$

In this equation, I is the current, R is the resistance, L is the inductance, and E is the electromotive force (voltage).

Solve the differential equation given a constant voltage E_0.

1. $I' + \frac{R}{L}I = \frac{E_0}{L}$
2. Integrating factor: $e^{\int (R/L)\,dt} = e^{Rt/L}$
3. $Ie^{Rt/L} = \int \frac{E_0}{L}e^{Rt/L}\,dt$
4. $\quad = \frac{E_0}{R}e^{Rt/L} + C$
5. $\quad I = \frac{E_0}{R} + Ce^{-Rt/L}$

51

Give the standard form of a first-order linear differential equation. What is its integrating factor?

1. $\frac{dy}{dx} + P(x)y = Q(x)$ Standard form
2. $u(x) = e^{\int P(x)\,dx}$ Integrating factor

Match the differential equation with its solution.

Differential Equation	*Solution*
$y' - 2x = 0$	(a) $y = Ce^{x^2}$
	(b) $y = -\frac{1}{2} + Ce^{x^2}$
	(c) $y = x^2 + C$
	(d) $y = Ce^{2x}$

1. $\int dy = \int 2x\,dx$
2. $y = x^2 + C$
3. Matches c.

53

Match the differential equation with its solution.

Differential Equation	*Solution*
$y' - 2x = 0$	(a) $y = Ce^{x^2}$
	(b) $y = -\frac{1}{2} + Ce^{x^2}$
	(c) $y = x^2 + C$
	(d) $y = Ce^{2x}$

1. $\int dy = \int 2x\,dx$
2. $y = x^2 + C$
3. Matches c.

55

Match the differential equation with its solution.

Differential Equation	*Solution*
$y' - 2xy = 0$	(a) $y = Ce^{x^2}$
	(b) $y = -\frac{1}{2} + Ce^{x^2}$
	(c) $y = x^2 + C$
	(d) $y = Ce^{2x}$

1. $\int \frac{dy}{y} = \int 2x\,dx$
2. $\ln y = x^2 + C_1$
3. $y = Ce^{x^2}$
4. Matches a.

57

Solve the first-order differential equation by any appropriate method.

$$\frac{dy}{dx} = \frac{e^{2x+y}}{e^{x-y}}$$

1. $e^{2x+y}\,dx - e^{x-y}\,dy = 0$
2. Separation of variables:

   $e^{2x}e^{y}\,dx = e^{x}e^{-y}\,dy$
3. $\int e^{x}\,dx = \int e^{-2y}\,dy$
4. $e^{x} = -\frac{1}{2}e^{-2y} + C_1$
5. $2e^{x} + e^{-2y} = C$

59

Solve the first-order differential equation by any appropriate method.

$$y \cos x - \cos x + \frac{dy}{dx} = 0$$

1. $(y \cos x - \cos x)\, dx + dy = 0$
2. Separation of variables:

$$\int \cos x \, dx = \int \frac{-1}{y-1}\, dy$$

3. $\sin x = -\ln(y - 1) + \ln C$
4. $\ln(y - 1) = -\sin x + \ln C$
5. $y = Ce^{-\sin x} + 1$

61

Solve the first-order differential equation by any appropriate method.

$$(3y^2 + 4xy)\, dx + (2xy + x^2)\, dy = 0$$

1. Homogeneous: $y = vx$
2. $dy = v\, dx + x\, dv$
3. $(3v^2x^2 + 4vx^2)\, dx + (2vx^2 + x^2)(v\, dx + x\, dv) = 0$
4. $\int \frac{5}{x}\, dx + \int \left(\frac{2v + 1}{v^2 + v}\right) dv = 0$
5. $\ln x^5 + \ln|v^2 + v| = \ln C$
6. $x^5(v^2 + v) = C$
7. $x^3y^2 + x^4y = C$

63

Solve the first-order differential equation by any appropriate method.

$(2y - e^{x})\,dx + x\,dy = 0$

1. Linear: $y' + \left(\frac{2}{x}\right)y = \frac{1}{x}e^{x}$
2. Integrating factor: $e^{\int (2/x)\,dx} = e^{\ln x^2}$
3. $= x^2$
4. $yx^2 = \int x^2 \frac{1}{x} e^{x}\,dx$
5. $= e^{x}(x - 1) + C$
6. $y = \frac{e^{x}}{x^2}(x - 1) + \frac{C}{x^2}$

65

Solve the first-order differential equation by any appropriate method.

$(x^2y^4 - 1)\,dx + x^3y^3\,dy = 0$

1. $y' + \left(\frac{1}{x}\right)y = x^{-3}y^{-3}$
2. Bernoulli: $n = -3$
3. $Q = x^{-3}$
4. $P = x^{-1}$
5. $e^{\int (4/x)\,dx} = e^{\ln x^4}$
6. $= x^4$
7. $y^4x^4 = \int 4(x^{-3})(x^4)\,dx$
8. $= 2x^2 + C$
9. $x^4y^4 - 2x^2 = C$

67

Solve the first-order differential equation by any appropriate method.

$3(y - 4x^2)\,dx + x\,dy = 0$

1. $3(y - 4x^2)\,dx = -x\,dy$
2. $x\frac{dy}{dx} = -3y + 12x^2$
3. $y' + \frac{3}{x}y = 12x$
4. Integrating factor: $e^{\int (3/x)\,dx} = e^{3 \ln x}$
5. $= x^3$
6. $y'x^3 + \frac{3}{x}x^3y = 12x(x^3)$
7. $= 12x^4$
8. $yx^3 = \int 12x^4\,dx$
9. $= \frac{12}{5}x^5 + C$
10. $y = \frac{12}{5}x^2 + \frac{C}{x^3}$

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Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

$y' + x\sqrt{y} = x^2$ is a first-order linear differential equation.

1. False
2. The equation contains $\sqrt{y}$.