Then use the Sum and Power Rules:

$$\frac{dy}{dx} = -x^{-2} - 3(-2)x^{-3} + 2(-3)x^{-4} = -\frac{1}{x^2} + \frac{6}{x^3} - \frac{6}{x^4}.$$

PROBLEMS

Find dy/dx in Problems 1–24.

1. $y = \frac{x^3}{3} - \frac{x^2}{2} + x - 1$
2. $y = (x-1)^3(x+2)^4$
3. $y = (x^2+1)^5$
4. $y = (x^3-3x)^4$
5. $y = (x+1)^2(x^2+1)^{-3}$
6. $y = \frac{2x+1}{x^2-1}$
7. $y = \frac{2x+5}{3x-2}$
8. $y = \left(\frac{x+1}{x-1}\right)^2$
9. $y = (1-x)(1+x^2)^{-1}$
10. $y = (x+1)^2(x^2+2x)^{-2}$
11. $y = \frac{5}{(2x-3)^4}$
12. $y = (x-1)^3(x+2)$
13. $y = (5-x)(4-2x)$
14. $y = [(5-x)(4-2x)]^2$
15. $y = (2x-1)^3(x+7)^{-3}$
16. $y = \frac{x^3+7}{x}$
17. $y = (2x^3-3x^2+6x)^{-5}$
18. $y = \frac{x^2}{(x-1)^2}$
19. $y = \frac{(x-1)^2}{x^2}$
20. $y = \frac{-1}{15(5x-1)^3}$
21. $y = \frac{12}{x} - \frac{4}{x^3} + \frac{3}{x^4}$
22. $y = \frac{(x-1)(x^2+x+1)}{x^3}$
23. $y = \frac{(x^2-1)}{x^2+x-2}$
24. $y = \frac{(x^2+x)(x^2-x+1)}{x^4}$

Find ds/dt in Problems 25–32.

25. $s = \frac{t}{t^2+1}$
26. $s = (2t+3)^3$
27. $s = (t^2-t)^{-2}$
28. $s = t^2(t+1)^{-1}$
29. $s = \frac{2t}{3t^2+1}$
30. $s = (t+t^{-1})^2$
31. $s = (t^2+3t)^3$
32. $s = (t^2-7t)(5-2t^3+t^4)/t^3$
33. Suppose that u and v are functions of x that are differentiable at $x = 0$ and that

$$u(0) = 5, \quad u'(0) = -3, \quad v(0) = -1, \quad v'(0) = 2.$$

Find the values of the derivatives below at $x = 0$.

a) $\frac{d}{dx}(uv)$ b) $\frac{d}{dx}\left(\frac{u}{v}\right)$ c) $\frac{d}{dx}\left(\frac{v}{u}\right)$

d) $\frac{d}{dx}(7v-2u)$ e) $\frac{d}{dx}(u^3)$ f) $\frac{d}{dx}(5v^{-3})$

34. Find an equation for the tangent to the curve $y = x/(x^2+1)$ at the origin.
35. Find an equation for the tangent to the curve $y = x + (1/x)$ at $x = 2$.
36. Find the first and second derivatives of

$$f(x) = (x^2+3x+1)^3.$$

37. Find d^2y/dx^2 if $y = (3-2x)^{-1}$.
38. Differentiate $x = 5y/(y+1)$ with respect to y.

To see how Eq. (3) works, calculate the derivatives of the functions in Problems 39–42 in two ways: (a) by applying Eq. (3) directly and (b) by first multiplying the factors to produce a polynomial to differentiate. In these examples, (b) is faster, but that is not always the case.

39. $y = x(x-1)(x+1)$
40. $y = (x-1)(x+1)(x^2+1)$
41. $y = (1-x)(x+1)(3-x^2)$
42. $y = x^2(x-1)(x^2+x+1)$
43. *Industrial production.* Economists often use the expression "rate of growth" in relative rather than absolute terms. For example, in a given industry, let $u = f(t)$ be the number of people in the labor force at time t. (This function will be treated as though it were differentiable even though it is an integer-valued step function. We approximate the step function by a smooth curve.)

   Let $v = g(t)$ be the average production per person in the labor force at time t. The total production is then $y = uv$. If the labor force is growing at the rate of 4 percent per year ($du/dt = 0.04u$) and the production per worker is growing at the rate of 5 percent per year ($dv/dt = 0.05v$), find the rate of growth of the total production, y.
44. Suppose that the labor force in Problem 43 is decreasing at the rate of 2 percent per year while the production per person is increasing at the rate of 3 percent per year. Is the total production increasing or decreasing, and at what rate?
45. *Rate of a chemical reaction.* When two chemicals, A and B, combine to form an amount p of product, the rate dp/dt at which the product forms is called the **reaction rate.** In many reactions, one molecule of product is formed from one molecule of A and one molecule of B. Suppose that the initial molar masses of A and B are equal, both having value a. Under these conditions, the amount of product at any time t after mixing is given by the function $p(t) = a^2kt/(akt+1)$. In this equation, k is a positive constant of

2.2 PRODUCTS, POWERS AND QUOTIENTS

1. $y = \frac{x^3}{3} - \frac{x^2}{2} + x - 1 \Rightarrow \frac{dy}{dx} = x^2 - x + 1$

3. $y = (x^2 + 1)^5 \Rightarrow$

$$\frac{dy}{dx} = 5(x^2+1)^4 \frac{d}{dx}(x^2+1) = 5(x^2+1)(2x) = 10x\ (x^2+1)^4$$

5. $y = (x+1)^2(x^2+1)^{-3} \Rightarrow$

$$\frac{dy}{dx} = (x+1)^2\left[-3(x^2+1)^{-4}\frac{d}{dx}(x^2+1)\right] + (x^2+1)^{-3}[2(x+1)]$$

$$= (x+1)^2(-6x)(x^2+1)^{-4} + 2(x+1)(x^2+1)^{-3}$$

$$= -2(x+1)(x^2+1)^{-4}(2x^2+3x-1)$$

-. $y = \dfrac{2x + 5}{3x - 2}$

$$\frac{dy}{dx} = \frac{(3x - 2)\frac{d}{dx}(2x + 5) - (2x + 5)\frac{d}{dx}(3x - 2)}{(3x - 2)^2}$$

$$= \frac{2(3x - 2) - 3(2x + 5)}{(3x - 2)^2}$$

$$= \frac{-19}{(3x - 2)^2}$$

9. $y = (1 - x)(1 + x^2)^{-1}$

$$\frac{dy}{dx} = (1 - x)\left[-(1 + x^2)^{-2}(2x)\right] + (1 + x^2)^{-1}(-1)$$

$$= (1 + x^2)^{-2}\left[-2x(1 - x) - (1 + x^2)\right]$$

$$= (1 + x^2)^{-2}(x^2 - 2x - 1)$$

11. $y = \dfrac{5}{(2x - 3)^4} = 5(2x - 3)^{-4} \Rightarrow \dfrac{dy}{dx} = -20(2x - 3)^{-5}(2) = \dfrac{-40}{(2x - 3)^5}$

13. $y = (5 - x)(4 - 2x) \Rightarrow \dfrac{dy}{dx} = -2(5 - x) - (4 - 2x) = 4x - 14$

15. $y = (2x - 1)^3(x + 7)^{-3}$

$$\frac{dy}{dx} = (2x - 1)^3\left[-3(x + 7)^{-4}\right] + (x + 7)^{-3}\left[3(2x - 1)^2(2)\right]$$

$$= (2x - 1)^2(x + 7)^{-4}\left[-3(2x - 1) + 6(x + 7)\right]$$

$$= 45(2x - 1)^2(x + 7)^{-4}$$

17. $y = (2x^3 - 3x^2 + 6x)^{-5} \Rightarrow \dfrac{dy}{dx} = -5(2x^3 - 3x^2 + 6x)^{-6}(6x^2 - 6x + 6)$

19. $y = \dfrac{(x - 1)^2}{x^2} = \dfrac{x^2 - 2x + 1}{x^2} = 1 - 2x^{-1} + x^{-2}$

$$\frac{dy}{dx} = 2x^{-2} - 2x^{-3}$$

21. $y = \dfrac{12}{x} - \dfrac{4}{x^3} + \dfrac{3}{x^4} = 12x^{-1} - 4x^{-3} + 3x^{-4}$

$$\frac{dy}{dx} = -12x^{-2} + 12x^{-4} - 12x^{-5}$$

23. $y = \dfrac{x^2 - 1}{x^2 + x - 2} = \dfrac{(x+1)(x-1)}{(x+2)(x-1)} = \dfrac{x+1}{x+2} \Rightarrow \dfrac{dy}{dx} = \dfrac{x + 2 - x - 1}{(x+2)^2} = \dfrac{1}{(x+2)^2}$

25. $s = \dfrac{t}{t^2 + 1} \Rightarrow \dfrac{ds}{dt} = \dfrac{t^2 + 1 - t(2t)}{(t^2 + 1)^2} = \dfrac{1 - t^2}{(t^2 + 1)^2}$

27. $s = (t^2 - t)^{-2} \Rightarrow \dfrac{ds}{dt} = -2(t^2 - t)^{-3}(2t - 1) = \dfrac{2 - 4t}{(t^2 - t)^3}$

29. $s = \dfrac{2t}{3t^2 + 1} \Rightarrow \dfrac{ds}{dt} = \dfrac{2(3t^2 + 1) - 2t(6t)}{(3t^2 + 1)^2} = \dfrac{2 - 6t^2}{(3t^2 + 1)^2}$

31. $s = (t^2 + 3t)^3 \Rightarrow \dfrac{ds}{dt} = 3(t^2 + 3t)^2(2t + 3)$

33. Each of the following is evaluated at $x = 0$:

(a) $\dfrac{d}{dx}(uv) = uv' + vu' = (5)(2) + (-1)(-3) = 13$

(b) $\dfrac{d}{dx}(\dfrac{u}{v}) = \dfrac{vu'-uv'}{v^2} = \dfrac{(-1)(-3)-(5)(2)}{(-1)^2} = -7$

(c) $\dfrac{d}{dx}(\dfrac{v}{u}) = \dfrac{uv'-vu'}{u^2} = \dfrac{(5)(2)-(-1)(-3)}{5^2} = \dfrac{7}{25}$

(d) $\dfrac{d}{dx}(7v-2u) = 7v'-2u' = 7(2) - 2(-3) = 20$

(e) $\dfrac{d}{dx}(u^3) = 3u^2u' = 3(5)^2(-3) = -225$

(f) $\dfrac{d}{dx}(5v^{-3}) = -3(5)(v^{-4})v' = -15(-1)^{-4}(2) = -30$

35. $y = x + \dfrac{1}{x} = x + x^{-1} \Rightarrow y' = 1 - x^{-2}$. When $x = 2$,

$y = 2 + \dfrac{1}{2} = \dfrac{5}{2}$ and $y' = 1 - \dfrac{1}{4} = \dfrac{3}{4}$.

$\therefore\ y - \dfrac{5}{2} = \dfrac{3}{4}(x - 2)$ or $4y - 3x = 4$.

37. $y = (3-2x)^{-1} \Rightarrow \dfrac{dy}{dx} = -(3-2x)^{-2}(-2) = 2(3-2x)^{-2}$

$\dfrac{d^2y}{dx^2} = -4(3-2x)^{-3}(-2) = 8(3-2x)^{-3}$

39. $y = x(x - 1)(x + 1)$

$\dfrac{dy}{dx} = x(x - 1)\dfrac{d}{dx}(x + 1) + x(x + 1)\dfrac{d}{dx}(x - 1) + (x + 1)(x - 1)\dfrac{d}{dx}(x)$

$= x(x - 1) + x(x + 1) + (x + 1)(x - 1) = 3x^2 - 1$

Alternatively, $y = x(x^2 - 1) = x^3 - x \Rightarrow \dfrac{dy}{dx} = 3x^2 - 1$