

Then use the Sum and Power Rules:

$$\frac{dy}{dx} = -x^{-2} - 3(-2)x^{-3} + 2(-3)x^{-4} = -\frac{1}{x^2} + \frac{6}{x^3} - \frac{6}{x^4}.$$

PROBLEMS

Find dy/dx in Problems 1–24.

1. $y = \frac{x^3}{3} - \frac{x^2}{2} + x - 1$
2. $y = (x-1)^3(x+2)^4$
3. $y = (x^2+1)^5$
4. $y = (x^3-3x)^4$
5. $y = (x+1)^2(x^2+1)^{-3}$
6. $y = \frac{2x+1}{x^2-1}$
7. $y = \frac{2x+5}{3x-2}$
8. $y = \left(\frac{x+1}{x-1}\right)^2$
9. $y = (1-x)(1+x^2)^{-1}$
10. $y = (x+1)^2(x^2+2x)^{-2}$
11. $y = \frac{5}{(2x-3)^4}$
12. $y = (x-1)^3(x+2)$
13. $y = (5-x)(4-2x)$
14. $y = [(5-x)(4-2x)]^2$
15. $y = (2x-1)^3(x+7)^{-3}$
16. $y = \frac{x^3+7}{x}$
17. $y = (2x^3-3x^2+6x)^{-5}$
18. $y = \frac{x^2}{(x-1)^2}$
19. $y = \frac{(x-1)^2}{x^2}$
20. $y = \frac{-1}{15(5x-1)^3}$
21. $y = \frac{12}{x} - \frac{4}{x^3} + \frac{3}{x^4}$
22. $y = \frac{(x-1)(x^2+x+1)}{x^3}$
23. $y = \frac{(x^2-1)}{x^2+x-2}$
24. $y = \frac{(x^2+x)(x^2-x+1)}{x^4}$

Find ds/dt in Problems 25–32.

25. $s = \frac{t}{t^2+1}$
26. $s = (2t+3)^3$
27. $s = (t^2-t)^{-2}$
28. $s = t^2(t+1)^{-1}$
29. $s = \frac{2t}{3t^2+1}$
30. $s = (t+t^{-1})^2$
31. $s = (t^2+3t)^3$
32. $s = (t^2-7t)(5-2t^3+t^4)/t^3$

33. Suppose that u and v are functions of x that are differentiable at $x=0$ and that

$$u(0) = 5, \quad u'(0) = -3, \quad v(0) = -1, \quad v'(0) = 2.$$

Find the values of the derivatives below at $x=0$.

- a) $\frac{d}{dx}(uv)$
- b) $\frac{d}{dx}\left(\frac{u}{v}\right)$
- c) $\frac{d}{dx}\left(\frac{v}{u}\right)$
- d) $\frac{d}{dx}(7v-2u)$
- e) $\frac{d}{dx}(u^3)$
- f) $\frac{d}{dx}(5v^{-3})$

34. Find an equation for the tangent to the curve $y = x/(x^2+1)$ at the origin.
35. Find an equation for the tangent to the curve $y = x + (1/x)$ at $x=2$.
36. Find the first and second derivatives of $f(x) = (x^2+3x+1)^3$.
37. Find d^2y/dx^2 if $y = (3-2x)^{-1}$.
38. Differentiate $x = 5y/(y+1)$ with respect to y .

To see how Eq. (3) works, calculate the derivatives of the functions in Problems 39–42 in two ways: (a) by applying Eq. (3) directly and (b) by first multiplying the factors to produce a polynomial to differentiate. In these examples, (b) is faster, but that is not always the case.

39. $y = x(x-1)(x+1)$
40. $y = (x-1)(x+1)(x^2+1)$
41. $y = (1-x)(x+1)(3-x^2)$
42. $y = x^2(x-1)(x^2+x+1)$

43. *Industrial production.* Economists often use the expression “rate of growth” in relative rather than absolute terms. For example, in a given industry, let $u = f(t)$ be the number of people in the labor force at time t . (This function will be treated as though it were differentiable even though it is an integer-valued step function. We approximate the step function by a smooth curve.)

Let $v = g(t)$ be the average production per person in the labor force at time t . The total production is then $y = uv$. If the labor force is growing at the rate of 4 percent per year ($du/dt = 0.04u$) and the production per worker is growing at the rate of 5 percent per year ($dv/dt = 0.05v$), find the rate of growth of the total production, y .

44. Suppose that the labor force in Problem 43 is decreasing at the rate of 2 percent per year while the production per person is increasing at the rate of 3 percent per year. Is the total production increasing or decreasing, and at what rate?
45. *Rate of a chemical reaction.* When two chemicals, A and B , combine to form an amount p of product, the rate dp/dt at which the product forms is called the **reaction rate**. In many reactions, one molecule of product is formed from one molecule of A and one molecule of B . Suppose that the initial molar masses of A and B are equal, both having value a . Under these conditions, the amount of product at any time t after mixing is given by the function $p(t) = a^2kt/(akt+1)$. In this equation, k is a positive constant of

2.2 PRODUCTS, POWERS AND QUOTIENTS

$$1. \quad y = \frac{x^3}{3} - \frac{x^2}{2} + x - 1 \Rightarrow \frac{dy}{dx} = x^2 - x + 1$$

$$3. \quad y = (x^2 + 1)^5 \Rightarrow$$

$$\frac{dy}{dx} = 5(x^2 + 1)^4 \frac{d}{dx}(x^2 + 1) = 5(x^2 + 1)(2x) = 10x(x^2 + 1)^4$$

$$5. \quad y = (x + 1)^2 (x^2 + 1)^{-3} \Rightarrow$$

$$\frac{dy}{dx} = (x + 1)^2 \left[-3(x^2 + 1)^{-4} \frac{d}{dx}(x^2 + 1) \right] + (x^2 + 1)^{-3} [2(x + 1)]$$

$$= (x + 1)^2 (-6x)(x^2 + 1)^{-4} + 2(x + 1)(x^2 + 1)^{-3}$$

$$= -2(x + 1)(x^2 + 1)^{-4}(2x^2 + 3x - 1)$$

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$$\begin{aligned}
 y &= \frac{2x+5}{3x-2} \Rightarrow \\
 \frac{dy}{dx} &= \frac{(3x-2) \frac{d}{dx}(2x+5) - (2x+5) \frac{d}{dx}(3x-2)}{(3x-2)^2} \\
 &= \frac{2(3x-2) - 3(2x+5)}{(3x-2)^2} \\
 &= \frac{-19}{(3x-2)^2}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad y &= (1-x)(1+x^2)^{-1} \Rightarrow \\
 \frac{dy}{dx} &= (1-x) [-(1+x^2)^{-2} (2x)] + (1+x^2)^{-1} (-1) \\
 &= (1+x^2)^{-2} [-2x(1-x) - (1+x^2)] \\
 &= (1+x^2)^{-2} (x^2 - 2x - 1)
 \end{aligned}$$

$$11. \quad y = \frac{5}{(2x-3)^4} = 5(2x-3)^{-4} \Rightarrow \frac{dy}{dx} = -20(2x-3)^{-5} (2) = \frac{-40}{(2x-3)^5}$$

$$13. \quad y = (5-x)(4-2x) \Rightarrow \frac{dy}{dx} = -2(5-x) - (4-2x) = 4x - 14$$

$$\begin{aligned}
 15. \quad y &= (2x-1)^3(x+7)^{-3} \Rightarrow \\
 \frac{dy}{dx} &= (2x-1)^3 [-3(x+7)^{-4}] + (x+7)^{-3} [3(2x-1)^2 (2)] \\
 &= (2x-1)^2 (x+7)^{-4} [-3(2x-1) + 6(x+7)] \\
 &= 45(2x-1)^2 (x+7)^{-4}
 \end{aligned}$$

$$17. \quad y = (2x^3 - 3x^2 + 6x)^{-5} \Rightarrow \frac{dy}{dx} = -5(2x^3 - 3x^2 + 6x)^{-6} (6x^2 - 6x + 6)$$

$$\begin{aligned}
 19. \quad y &= \frac{(x-1)^2}{x^2} = \frac{x^2 - 2x + 1}{x^2} = 1 - 2x^{-1} + x^{-2} \\
 \frac{dy}{dx} &= 2x^{-2} - 2x^{-3} \Rightarrow
 \end{aligned}$$

$$\begin{aligned}
 21. \quad y &= \frac{12}{x} - \frac{4}{x^3} + \frac{3}{x^4} = 12x^{-1} - 4x^{-3} + 3x^{-4} \\
 \frac{dy}{dx} &= -12x^{-2} + 12x^{-4} - 12x^{-5}
 \end{aligned}$$

$$23. \quad y = \frac{x^2-1}{x^2+x-2} = \frac{(x+1)(x-1)}{(x+2)(x-1)} = \frac{x+1}{x+2} \Rightarrow \frac{dy}{dx} = \frac{x+2-x-1}{(x+2)^2} = \frac{1}{(x+2)^2}$$

$$25. \quad s = \frac{t}{t^2+1} \Rightarrow \frac{ds}{dt} = \frac{t^2+1-t(2t)}{(t^2+1)^2} = \frac{1-t^2}{(t^2+1)^2}$$

$$27. s = (t^2 - t)^{-2} \Rightarrow \frac{ds}{dt} = -2(t^2 - t)^{-3} (2t - 1) = \frac{2 - 4t}{(t^2 - t)^3}$$

$$29. s = \frac{2t}{3t^2 + 1} \Rightarrow \frac{ds}{dt} = \frac{2(3t^2 + 1) - 2t(6t)}{(3t^2 + 1)^2} = \frac{2 - 6t^2}{(3t^2 + 1)^2}$$

$$31. s = (t^2 + 3t)^3 \Rightarrow \frac{ds}{dt} = 3(t^2 + 3t)^2 (2t + 3)$$

33. Each of the following is evaluated at $x = 0$:

$$(a) \frac{d}{dx}(uv) = uv' + vu' = (5)(2) + (-1)(-3) = 13$$

$$(b) \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2} = \frac{(-1)(-3) - (5)(2)}{(-1)^2} = -7$$

$$(c) \frac{d}{dx}\left(\frac{v}{u}\right) = \frac{uv' - vu'}{u^2} = \frac{(5)(2) - (-1)(-3)}{5^2} = \frac{7}{25}$$

$$(d) \frac{d}{dx}(7v - 2u) = 7v' - 2u' = 7(2) - 2(-3) = 20$$

$$(e) \frac{d}{dx}(u^3) = 3u^2 u' = 3(5)^2(-3) = -225$$

$$(f) \frac{d}{dx}(5v^{-3}) = -3(5)(v^{-4})v' = -15(-1)^{-4}(2) = -30$$

$$35. y = x + \frac{1}{x} = x + x^{-1} \Rightarrow y' = 1 - x^{-2}. \text{ When } x = 2,$$

$$y = 2 + \frac{1}{2} = \frac{5}{2} \text{ and } y' = 1 - \frac{1}{4} = \frac{3}{4}.$$

$$\therefore y - \frac{5}{2} = \frac{3}{4}(x - 2) \text{ or } 4y - 3x = 4.$$

$$37. y = (3-2x)^{-1} \Rightarrow \frac{dy}{dx} = -(3-2x)^{-2}(-2) = 2(3-2x)^{-2}$$

$$\frac{d^2y}{dx^2} = -4(3-2x)^{-3}(-2) = 8(3-2x)^{-3}$$

$$39. y = x(x-1)(x+1)$$

$$\frac{dy}{dx} = x(x-1)\frac{d}{dx}(x+1) + x(x+1)\frac{d}{dx}(x-1) + (x+1)(x-1)\frac{d}{dx}(x)$$

$$= x(x-1) + x(x+1) + (x+1)(x-1) = 3x^2 - 1$$

$$\text{Alternatively, } y = x(x^2 - 1) = x^3 - x \Rightarrow \frac{dy}{dx} = 3x^2 - 1$$