

Properties of $y = e^x$

1. The exponential function $y = e^x$ is the inverse of the natural logarithm function $y = \ln x$; that is, $e^x = \ln^{-1}x$.

Domain: The set of all real numbers, $-\infty < x < \infty$.

Range: The set of all positive numbers, $y > 0$.

2. Its derivative is

$$\frac{d}{dx}(e^x) = e^x.$$

3. It is continuous (because it is differentiable) and is an increasing function of x .

4. If u is any differentiable function of x , then

$$\frac{d}{dx}e^u = e^u \frac{du}{dx} \quad \text{and} \quad \int e^u du = e^u + C.$$

5. $e^{x_1} \cdot e^{x_2} = e^{x_1+x_2}$ and $e^{-x} = 1/e^x$

PROBLEMS

Simplify the expressions in Problems 1–12.

- | | | |
|------------------------|-------------------|---------------------------|
| 1. $e^{\ln x}$ | 2. $\ln(e^x)$ | 3. $e^{-\ln(x^2)}$ |
| 4. $\ln(e^{-x^2})$ | 5. $\ln(e^{1/x})$ | 6. $\ln(1/e^x)$ |
| 7. $e^{\ln 2 + \ln x}$ | 8. $e^{2 \ln x}$ | 9. $\ln(e^{x-x^2})$ |
| 10. $\ln(x^2 e^{-2x})$ | 11. $e^{x+\ln x}$ | 12. $e^{\ln x - 2 \ln y}$ |

In Problems 13–18, solve for y .

13. $e^{\sqrt{y}} = x^2$
 14. $e^{2y} = x^2$
 15. $e^{(x^2)} \cdot e^{(2x+1)} = e^y$
 16. $\ln(y-1) = x + \ln x$
 17. $\ln(y-2) = \ln(\sin x) - x$
 18. $\ln(y^2 - 1) - \ln(y + 1) = \sin x$

Find dy/dx in Problems 19–42.

- | | |
|---------------------------------|-----------------------------------|
| 19. $y = e^{3x}$ | 20. $y = e^{(x+1)}$ |
| 21. $y = e^{5-7x}$ | 22. $y = \cos e^x$ |
| 23. $y = x^2 e^x$ | 24. $y = \sin e^{-x}$ |
| 25. $y = e^{\sin x}$ | 26. $y = e^{(x^2)} \cdot e^{-x}$ |
| 27. $y = \ln(3xe^{-x})$ | 28. $y = \ln \frac{e^x}{1+e^x}$ |
| 29. $y = e^{\sin^{-1}x}$ | 30. $y = (1+2x)e^{-2x}$ |
| 31. $y = (9x^2 - 6x + 2)e^{3x}$ | 32. $y = \frac{ax-1}{a^2} e^{ax}$ |
| 33. $y = x^2 e^{-x^2}$ | 34. $y = e^x \ln x$ |
| 35. $y = \tan^{-1}(e^x)$ | 36. $y = \sec^{-1}(e^{2x})$ |

37. $y = x^3 e^{-2x} \cos 5x$ (Hint: Use logarithmic differentiation.)

38. $y = \int_0^{\ln x} \sin e^t dt, \quad x > 0$
 39. $\ln y = x \sin x$
 40. $\ln xy = e^{x+y}$
 41. $e^{2x} = \sin(x+3y)$
 42. $\tan y = e^x + \ln x$

Evaluate the integrals in Problems 43–56.

- | | |
|------------------------------------------------------------------------------|---------------------------------------|
| 43. $\int_{\ln 3}^{\ln 5} e^{2x} dx$ | 44. $\int_{-1}^1 x e^{x^2} dx$ |
| 45. $\int_0^{\pi} e^{\sin x} \cos x dx$ | 46. $\int_0^{\ln 8} e^{x/3} dx$ |
| 47. $\int_{-\ln(a+1)}^0 e^{-x} dx$ | 48. $\int_0^2 e^{x/2} dx$ |
| 49. $\int_0^1 e^{\ln \sqrt{x}} dx$ | 50. $\int_0^1 \frac{dx}{e^x}$ |
| 51. $\int_0^{\ln 2} \frac{24 dx}{e^{3x}}$ | 52. $\int_0^1 \frac{e^x dx}{1+e^x}$ |
| 53. $\int_0^{\ln 13} \frac{e^x dx}{1+2e^x}$ | 54. $\int_e^{e^2} \frac{dx}{x \ln x}$ |
| 55. $\int_0^{\ln 2} \frac{e^x dx}{1+e^{2x}}$ (Hint: Let $u = e^x$.) | |
| 56. $\int_1^4 \frac{e^{\sqrt{x}} dx}{\sqrt{x}}$ (Hint: Let $u = \sqrt{x}$.) | |

gives a way to define e independently of the definition of the natural logarithm. However, the proof that this limit exists would then have to be different from the proof here which uses the logarithm. □

Properties of $y = a^x$, $a > 0$, $a \neq 1$

If a is a positive real number and $a \neq 1$, then the function $y = a^x$ has the following properties.

1. It is defined by the equation

$$a^x = e^{x \ln a}.$$

Domain: the set of all real numbers, $-\infty < x < \infty$.

Range: the set of all positive real numbers, $y > 0$.

2. Its derivative is

$$\frac{d}{dx} a^x = a^x \ln a.$$

3. It is continuous (because it is differentiable), increasing if $a > 1$, decreasing if $0 < a < 1$, and one-to-one in either case.

4. If u is any differentiable function of x , then

$$\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx} \quad \text{and} \quad \int a^u du = \frac{1}{\ln a} a^u + C.$$

PROBLEMS

Problems 1–16, find dy/dx .

1. $y = 2^x$
2. $y = 2^{3x}$
3. $y = 8^x$
4. $y = 3^{2x}$
5. $y = 9^x$
6. $y = 2^{x^3}$
7. $y = (2^x)^2$
8. $y = x^{\sin x}$, $x > 0$
9. $y = (\sin x)^{\tan x}$, $\sin x > 0$
10. $y = 2^{\sec x}$
11. $y = x^{\ln x}$, $x > 0$
12. $y = (\cos x)^x$, $\cos x > 0$
13. $y = (1-x)^x$, $x < 1$
14. $y = x^{2(x^2)}$
15. $y = 2^x \ln x$
16. $y = (\cos x)^{\sqrt{x}}$, $x > 0$, $\cos x > 0$

Evaluate the integrals in Problems 17–28.

17. $\int_0^1 5^x dx$
18. $\int_{-1}^0 2^x dx$
19. $\int_0^1 \frac{1}{2^x} dx$
20. $\int_{-1}^1 \left(\frac{1}{10}\right)^x dx$

21. $\int_0^1 3^{2x} dx$
22. $\int_{-1}^1 2^{(x+1)} dx$
23. $\int_{-1}^0 4^{-x} \ln 2 dx$
24. $\int_{-2}^0 5^{-x} dx$
25. $\int_1^2 5^{(2x-2)} dx$
26. $\int_1^{\sqrt{2}} x 2^{-x^2} dx$
27. $\int_0^{\pi/2} 2^{\cos x} \sin x dx$
28. $\int_0^{\pi/3} 2^{\sec x} \sec x \tan x dx$

29. Which integral has the larger value: a, or b?

a) $\int_0^1 2^{(3x)} dx$ b) $\int_0^1 3^{(2x)} dx$

30. Find the derivative with respect to x of the following functions.

- a) $y = 2^{\ln x}$ b) $y = \ln 2^x$
 c) $y = \ln x^2$ d) $y = (\ln x)^2$

Find the limits in Problems 31–36.

31. $\lim_{x \rightarrow \infty} 2^{-x}$
32. $\lim_{x \rightarrow -\infty} 3^x$

6.6 THE EXPONENTIAL FUNCTION e^x

1. $e^{\ln x} = x$

3. $e^{-\ln(x^2)} = e^{\ln(x^{-2})} = x^{-2}$

5. $\ln e^{\frac{1}{x}} = \frac{1}{x}$

7. $e^{\ln 2 + \ln x} = e^{\ln 2x} = 2x$

9. $\ln e^{(x-x^2)} = x - x^2$

11. $e^{(x+\ln x)} = e^x e^{\ln x} = xe^x$

13. $e^{\sqrt{y}} = x^2 \Rightarrow \sqrt{y} \ln e = \ln x^2 \Rightarrow \sqrt{y} = 2 \ln x \Rightarrow y = 4 \ln^2 x$

15. $e^{(x^2)} e^{2x+1} = e^y \Leftrightarrow y = x^2 + 2x + 1$

17. $\ln(y-2) = \ln(\sin x) - x \Rightarrow \ln \frac{y-2}{\sin x} = -x \Rightarrow \frac{y-2}{\sin x} = e^{-x}$

$y = e^{-x} \sin x + 2.$

19. $y = e^{3x} \Rightarrow \frac{dy}{dx} = 3e^{3x}$

21. $y = e^{5-7x} \Rightarrow \frac{dy}{dx} = -7e^{5-7x}$

23. $y = x^2 e^x = \frac{dy}{dx} = x^2 e^x + 2xe^x = xe^x(x+2)$

25. $y = e^{\sin x} \Rightarrow \frac{dy}{dx} = (\cos x)e^{\sin x}$

27. $y = \ln(3xe^{-x}) = \ln 3 + \ln x - x \Rightarrow \frac{dy}{dx} = \frac{1}{x} - 1 = \frac{1-x}{x}$

29. $y = e^{\sin^{-1} x} \Rightarrow \frac{dy}{dx} = e^{\sin^{-1} x} \left(\frac{1}{\sqrt{1-x^2}} \right)$

31. $y = (9x^2 - 6x + 2)e^{3x} \Rightarrow \frac{dy}{dx} = 3(9x^2 - 6x + 2)e^{3x} + (18x - 6)e^{3x} = 27x^2 e^{3x}$

33. $y = x^2 e^{-(x^2)} \Rightarrow \frac{dy}{dx} = x^2 (-2xe^{-(x^2)}) + 2xe^{-(x^2)} = 2xe^{-(x^2)}(1-x^2)$

35. $y = \tan^{-1}(e^x) \Rightarrow \frac{dy}{dx} = \frac{e^x}{1+e^{2x}}$

37. $y = x^3 e^{-2x} \cos 5x \Rightarrow \ln y = 3 \ln x - 2x + \ln(\cos 5x)$
 $\frac{1}{y} \frac{dy}{dx} = \frac{3}{x} - 2 - \frac{5 \sin 5x}{\cos 5x} = \frac{3}{x} - 2 - 5 \tan 5x$
 $\frac{dy}{dx} = y \left(\frac{3}{x} - 2 - 5 \tan 5x \right)$

39. $\ln y = x \sin x \Rightarrow \frac{1}{y} \frac{dy}{dx} = \sin x + x \cos x \Rightarrow \frac{dy}{dx} = y(\sin x + x \cos x)$

41. $e^{2x} = \sin(x+3y) \Rightarrow 2e^{2x} = \cos(x+3y) \left[1 + 3 \frac{dy}{dx} \right]$
 $2e^{2x} = \cos(x+3y) + 3\cos(x+3y) \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{2e^{2x} - \cos(x+3y)}{3\cos(x+3y)}$$

43. $\int_{\ln 3}^{\ln 5} e^{2x} dx = \left. \frac{1}{2} e^{2x} \right|_{\ln 3}^{\ln 5} = \frac{1}{2} [e^{2 \ln 5} - e^{2 \ln 3}] = \frac{1}{2}(25 - 9) = 8$

45. $\int_0^{\pi} e^{\sin x} \cos x dx = \left. e^{\sin x} \right|_0^{\pi} = 0$

47. $\int_{-\ln(a+1)}^0 e^{-x} dx = \left. -e^{-x} \right|_{-\ln(a+1)}^0 = -[e^0 - e^{\ln(a+1)}] = a$

49. $\int_0^1 e^{\ln \sqrt{x}} dx = \int_0^1 \sqrt{x} dx = \left. \frac{2}{3} x^{3/2} \right|_0^1 = \frac{2}{3}$

6.6 The Exponential Function e^x

$$51. \int_0^{\ln 2} \frac{24 dx}{e^{3x}} = -8 e^{-3x} \Big|_0^{\ln 2} = -8 (e^{-3 \ln 2} - e^0) = -8 \left(\frac{1}{8} - 1 \right) = 7$$

$$53. \int_0^{\ln 13} \frac{e^x dx}{1+2e^x} = \frac{1}{2} \ln(1+2e^x) \Big|_0^{\ln 13} = \frac{1}{2} (\ln 27 - \ln 3) = \ln 3$$

$$55. \int_0^{\ln 2} \frac{e^x}{1+e^{2x}} dx = \tan^{-1} e^x \Big|_0^{\ln 2} = \tan^{-1} 2 - \frac{\pi}{4}$$

$$57. \lim_{h \rightarrow 0} \frac{e^h - (1+h)}{h^2} = \lim_{h \rightarrow 0} \frac{e^h - 1}{2h} = \lim_{h \rightarrow 0} \frac{e^h}{2} = \frac{1}{2}$$

$$59. \lim_{x \rightarrow \infty} \frac{x^2 + e^x}{x + e^x} = \lim_{x \rightarrow \infty} \frac{2x + e^x}{1 + e^x} = \lim_{x \rightarrow \infty} \frac{2 + e^x}{e^x} = \lim_{x \rightarrow \infty} \frac{e^x}{e^x} = 1$$

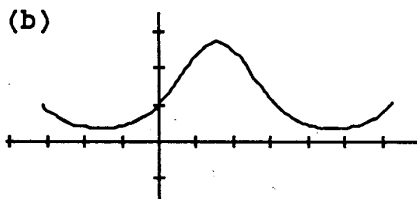
$$61. (a) y = e^{\sin x}, -\pi \leq x \leq 2\pi. y'(x) = (\cos x)e^{\sin x} = 0 \Leftrightarrow \cos x = 0.$$

$$\therefore x = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}. y\left(\frac{\pi}{2}\right) = e^{\sin\left(\frac{\pi}{2}\right)} = e. y\left(-\frac{\pi}{2}\right) = y\left(\frac{3\pi}{2}\right) = \frac{1}{e}.$$

$$\text{Testing endpoints: } y(\pi) = e^{\sin \pi} = 1 \text{ and } y(2\pi) = 1.$$

$$\text{The absolute maximum} = e \text{ at } x = \frac{\pi}{2} \text{ and the}$$

$$\text{absolute minimum} = \frac{1}{e} \text{ at } x = -\frac{\pi}{2} \text{ or } \frac{3\pi}{2}.$$



$$63. f(x) = x^2 \ln \frac{1}{x} = -x^2 \ln x \Rightarrow f'(x) = x^2 \left(\frac{1}{x} \right) - 2x \ln x = 0 \text{ if}$$

$$-x(1 + 2 \ln x) = 0 \Leftrightarrow x = 0 \text{ or } x = e^{-1/2}. f''(x) = -3 - 2 \ln x$$

$$\text{and } f''(e^{-1/2}) = -2 \Rightarrow f(e^{-1/2}) = \frac{1}{2e} \text{ is a maximum.}$$

$$65. (a) f(x) = f'(x) = f''(x) = f'''(x) = e^x$$

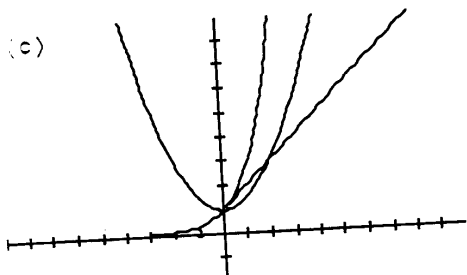
$$\text{Linear: } e^x \approx f(0) + f'(0)x = 1 + x$$

$$\text{Quadratic: } e^x \approx f(0) + f'(0)x + \frac{f''(0)}{2}x^2 = 1 + x + \frac{1}{2}x^2$$

$$(b) |e_1(x)| \leq \frac{1}{2} Mx^2 = \frac{1}{2} (1)(0.1)^2 = 0.005$$

$$|e_2(x)| \leq \frac{1}{6} Mx^3 = \frac{1}{6} (1)(0.1)^3 \approx 0.00017$$

Chapter 6: Transcendental Functions



7. $A(t) = \int_0^t e^{-x} dx = -e^{-x} \Big|_0^t = -e^{-t} + 1$

$$V(t) = \pi \int_0^t (e^{-x})^2 dx = \pi \left[-\frac{1}{2} e^{-2x} \right]_0^t = \frac{\pi}{2} (1 - e^{-2t})$$

(a) $\lim_{t \rightarrow \infty} A(t) = \lim_{t \rightarrow \infty} (-e^{-t} + 1) = 1$

(b) $\lim_{t \rightarrow \infty} \frac{V(t)}{A(t)} = \lim_{t \rightarrow \infty} \frac{\pi \left(\frac{1 - e^{-2t}}{-e^{-t} + 1} \right)}{2} = \frac{\pi}{2}$

(c) $\lim_{t \rightarrow 0^+} \frac{V(t)}{A(t)} = \lim_{t \rightarrow 0^+} \frac{\pi \left(\frac{1 - e^{-2t}}{-e^{-t} + 1} \right)}{2} = \lim_{t \rightarrow 0^+} \frac{\pi}{2} \left(\frac{2e^{-2t}}{e^{-t}} \right) = \pi$

69. (a) $y = Ce^{ax} \Rightarrow \frac{dy}{dx} = aCe^{ax} = ay$

(b) $\frac{dy}{dt} = -2y \Rightarrow y = Ce^{-2t}$. $3 = Ce^0 \Rightarrow C = 3$. $\therefore y = 3e^{-2t}$

71. $\frac{dy}{dx} = e^{-x}$, $y = 0$ when $x = 4 \Rightarrow y = -e^{-x} + C$.

$$0 = -e^{-4} + C \Rightarrow C = e^{-4} \therefore y = -e^{-x} + e^{-4}$$

73. $\frac{1}{y+1} \frac{dy}{dx} = \frac{1}{2x}$, $x > 0$, $y = 1$ when $x = 2$

$$\frac{dy}{y+1} = \frac{dx}{2x} \Rightarrow \ln|y+1| = \frac{1}{2} \ln|x| + C$$

$$C = \ln 2 - \ln \sqrt{2} = \ln \frac{2}{\sqrt{2}} = \ln \sqrt{2}$$

$$\ln|1+y| = \ln \sqrt{x} + \ln \sqrt{2} = \ln \sqrt{2x} \Rightarrow y+1 = \sqrt{2x} \text{ or } (y+1)^2 = 2x$$

75. $\frac{d}{dx}(\cosh x) = \frac{d}{dx} \left[\frac{1}{2}(e^x + e^{-x}) \right] = \frac{1}{2}(e^x - e^{-x}) = \sinh x$

77. $\cosh(-x) = \frac{1}{2}(e^{-x} + e^x) = \frac{1}{2}(e^x + e^{-x}) = \cosh x$

$$\sinh(-x) = \frac{1}{2}(e^{-x} - e^x) = -\frac{1}{2}(e^x - e^{-x}) = -\sinh x$$

6.7 THE FUNCTIONS a^x and a^u

1. If $y = 2^x = e^{\ln 2^x} = e^{x \ln 2}$, then $\frac{dy}{dx} = (\ln 2) e^{x \ln 2} = (\ln 2) 2^x$
 or $y = 2^x \Rightarrow \ln y = \ln 2^x = x \ln 2$, so $\frac{1}{y} \frac{dy}{dx} = \ln 2$, $\frac{dy}{dx} = y \ln 2 = (\ln 2) 2^x$
3. $y = 8^x = e^{\ln 8^x} = e^{x \ln 8} \Rightarrow \frac{dy}{dx} = (\ln 8) e^{x \ln 8} = (\ln 8) 8^x$
5. $y = 9^x = e^{\ln 9^x} = e^{x \ln 9} \Rightarrow \frac{dy}{dx} = (\ln 9) e^{x \ln 9} = (\ln 9) 9^x$
7. $y = (2^x)^2 = (2^2)^x = 4^x$. $\frac{dy}{dx} = (\ln 4) 4^x$
9. $y = (\sin x)^{\tan x}$, $\sin x > 0 \Rightarrow \ln y = (\tan x) \ln (\sin x)$
 $\frac{1}{y} \frac{dy}{dx} = (\tan x) \left(\frac{\cos x}{\sin x} \right) + [\ln (\sin x)] (\sec^2 x)$
 $\frac{dy}{dx} = y [1 + (\sec^2 x) \ln (\sin x)]$
11. $y = x^{\ln x}$, $x > 0 \Rightarrow \ln y = (\ln x)(\ln x) = \ln^2 x$
 $\frac{1}{y} \frac{dy}{dx} = 2 \ln x \left(\frac{1}{x} \right) \Rightarrow \frac{dy}{dx} = 2y \left(\frac{\ln x}{x} \right) = \frac{2 \ln x (x^{\ln x})}{x}$
13. $y = (1-x)^x$, $x < 1 \Rightarrow \ln y = x \ln (1-x)$
 $\frac{1}{y} \frac{dy}{dx} = \frac{-x}{1-x} + \ln (1-x) \Rightarrow \frac{dy}{dx} = y \left(\frac{-x}{1-x} + \ln (1-x) \right)$ or
 $\frac{dy}{dx} = (1-x)^x \left(\frac{-x}{1-x} + \ln (1-x) \right)$
15. $y = 2^x \ln x \Rightarrow \frac{dy}{dx} = 2^x \left(\frac{1}{x} \right) + \ln x (\ln 2) 2^x = 2^x \left[\frac{1}{x} + \ln 2 (\ln x) \right]$
17. $\int_0^1 5^x dx = \left[\frac{1}{\ln 5} 5^x \right]_0^1 = \frac{1}{\ln 5} (5 - 1) = \frac{4}{\ln 5}$
19. $\int_0^1 \frac{1}{2^x} dx = \int_0^1 2^{-x} dx = \left[-\frac{1}{\ln 2} 2^{-x} \right]_0^1 = -\frac{1}{\ln 2} \left[\frac{1}{2} - 1 \right] = \frac{1}{2 \ln 2}$
21. $\int_0^1 3^{2x} dx = \left[\frac{1}{2 \ln 3} 3^{2x} \right]_0^1 = \frac{1}{2 \ln 3} [3^2 - 1] = \frac{4}{\ln 3}$
23. $\int_{-1}^0 4^{-x} \ln 2 dx = \left[-\frac{\ln 2}{\ln 4} 4^{-x} \right]_{-1}^0 = -\frac{\ln 2}{\ln 4} [1 - 4] = \frac{3 \ln 2}{\ln 4}$
25. $\int_1^2 5^{(2x-2)} dx = \left[\frac{1}{2 \ln 5} 5^{(2x-2)} \right]_1^2 = \frac{1}{2 \ln 5} [5^2 - 5^0] = \frac{24}{\ln 25}$
27. $\int_0^{\frac{\pi}{2}} 2^{\cos x} \sin x dx = \left[-\frac{1}{\ln 2} 2^{\cos x} \right]_0^{\frac{\pi}{2}} = -\frac{1}{\ln 2} [2^0 - 2] = \frac{1}{\ln 2}$

29. (b), because $\int_0^1 2^{3x} dx = \int_0^1 8^x dx$, and $\int_0^1 3^{2x} dx = \int_0^1 9^x dx$

31. $\lim_{x \rightarrow \infty} 2^{-x} = 0$

33. $\lim_{x \rightarrow 0} \frac{3^{\sin x} - 1}{x} = \lim_{x \rightarrow 0} \frac{3^{\sin x} (\cos x) (\ln 3)}{1} = \ln 3$

35. Let $y = (e^x + x)^{\frac{1}{x}}$ so that $\ln y = \frac{1}{x} \ln(e^x + x)$. We first find

$$\lim_{x \rightarrow 0} \frac{\ln(e^x + x)}{x} = \lim_{x \rightarrow 0} \frac{e^x + 1}{e^x + x} = 2. \text{ Thus } \lim_{x \rightarrow 0} \ln y = 2, \text{ so}$$

$$\lim_{x \rightarrow 0} e^{\ln y} = \lim_{x \rightarrow 0} (e^x + x)^{\frac{1}{x}} = e^2.$$

37. (a) $\lim_{x \rightarrow \infty} \frac{3^x - 5}{4(3^x + 2)} = \lim_{x \rightarrow \infty} \frac{1 - \frac{5}{3^x}}{4\left(1 + \frac{2}{3^x}\right)} = \frac{1}{4}$

(b) $\lim_{x \rightarrow \infty} \frac{3^x - 5}{4(3^x + 2)} = \frac{0 - 5}{4(0 + 2)} = -\frac{5}{8}$

39. Let $f(x) = 2^x - x^2$, so that $f'(x) = (\ln 2)2^x - 2x$. Let $x_0 = -0.5$.

Using the iterative formula $x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$, we find

$$x = -0.766664696, y = 0.587774756.$$

41. Let $y = x^{(1/x^n)}$ and consider $\ln y = \frac{1}{x^n} \ln x$.

$$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \frac{\ln x}{x^n} = \lim_{n \rightarrow \infty} \frac{\frac{1}{x}}{nx^{n-1}} = \lim_{n \rightarrow \infty} \frac{1}{nx^n} = 0. \text{ Hence}$$

$$\lim_{x \rightarrow \infty} e^{\ln y} = \lim_{x \rightarrow \infty} x^{(1/x^n)} = e^0 = 1$$

43. Equation 2 implies line (a), commutativity of multiplication implies line (b), Equation 4 implies line (c), and Equation 2 implies line (d).