In Problems 1–29, find dy/dx.

1. $x^2 + y^2 = 1$
2. $y^2 = \dfrac{x-1}{x+1}$
3. $x^2 - xy = 2$
4. $x^2y + xy^2 = 6$
5. $y^2 = x^3$
6. $x^{2/3} + y^{2/3} = 1$
7. $x^{1/2} + y^{1/2} = 1$
8. $x^3 - xy + y^3 = 1$
9. $x^2 = \dfrac{x-y}{x+y}$
10. $y = \dfrac{x}{\sqrt{x^2+1}}$
11. $y = x\sqrt{x^2+1}$
12. $y^2 = x^2 + \dfrac{1}{x^2}$
13. $2xy + y^2 = x + y$
14. $y = \sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}$
15. $y^2 = \dfrac{x^2-1}{x^2+1}$
16. $(x+y)^3 + (x-y)^3 = x^4 + y^4$
17. $(3x+7)^5 = 2y^3$
18. $y = (x+5)^4(x^2-2)^3$
19. $\dfrac{1}{y} + \dfrac{1}{x} = 1$
20. $y = (x^2+5x)^3$
21. $y^2 = x^2 - x$
22. $x^2y^2 = x^2 + y^2$
23. $y = \dfrac{\sqrt[3]{x^2+3}}{x}$
24. $y = x^2\sqrt{1-x^2}$
25. $x^3 + y^3 = 18xy$
26. $y = (3x^2+5x+1)^{3/2}$
27. $y = (2x+5)^{-1/5}$
28. $y = 3(2x^{-1/2}+1)^{-1/3}$
29. $y = \sqrt{1-\sqrt{x}}$
30. Find dT/dL if $T^2 = 4\pi^2 L/g$. (This equation gives the period T of a simple pendulum in terms of its length L and the acceleration of gravity g.)
31. Find the x- and y-intercepts of the line tangent to the curve $y = x^{1/2}$ at $x = 4$.

2.3 IMPLICIT DIFFERENTIATION AND FRACTIONAL POWERS

1. $x^2 + y^2 = 1 \Rightarrow 2x + 2y\dfrac{dy}{dx} = 0 \Rightarrow 2y\dfrac{dy}{dx} = -2x \Rightarrow \dfrac{dy}{dx} = -\dfrac{x}{y}$

3. $x^2 - xy = 2 \Rightarrow 2x - x\dfrac{dy}{dx} - y = 0 \Rightarrow -x\dfrac{dy}{dx} = y - 2x \Rightarrow$

$$\frac{dy}{dx} = \frac{y-2x}{x}$$

5. $y^2 = x^3 \Rightarrow 2y\dfrac{dy}{dx} = 3x^2 \Rightarrow \dfrac{dy}{dx} = \dfrac{3x^2}{2y}$

7. $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 1 \Rightarrow \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$

$$y^{-\frac{1}{2}}\frac{dy}{dx} = -x^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = -x^{-\frac{1}{2}}y^{\frac{1}{2}} = -\sqrt{\frac{y}{x}}$$

9. $x^2 = \frac{x-y}{x+y} \Rightarrow x^3 + x^2y = x - y \Rightarrow 3x^2 + x^2\frac{dy}{dx} + 2xy = 1 - \frac{dy}{dx}$

$$(x^2 + 1)\frac{dy}{dx} = 1 - 2xy - 3x^2 \Rightarrow \frac{dy}{dx} = \frac{1 - 2xy - 3x^2}{x^2 + 1}$$

11. $y = x\sqrt{x^2+1} = x(x^2+1)^{\frac{1}{2}}$

$$\frac{dy}{dx} = (x^2+1)^{\frac{1}{2}} + x\left[\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)\right] = \frac{\sqrt{x^2+1}}{1} + \frac{x^2}{\sqrt{x^2+1}} = \frac{2x^2+1}{\sqrt{x^2+1}}$$

13. $2xy + y^2 = x + y \Rightarrow 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$$2x\frac{dy}{dx} + 2y\frac{dy}{dx} - \frac{dy}{dx} = 1 - 2y$$

$$\frac{dy}{dx} = \frac{1 - 2y}{2x + 2y - 1}$$

15. $y^2 = \frac{x^2 - 1}{x^2 + 1} \Rightarrow 2y\frac{dy}{dx} = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}$

$$2y\frac{dy}{dx} = \frac{2x^3 + 2x - 2x^3 + 2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2} \Rightarrow \frac{dy}{dx} = \frac{2x}{y(x^2+1)^2}$$

17. $(3x + 7)^5 = 2y^3 \Rightarrow 5(3x+7)^4(3) = 6y^2\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{5(3x+7)^4}{2y^2}$$

19. $\frac{1}{x} + \frac{1}{y} = 1 \Rightarrow x^{-1} + y^{-1} = 1 \Rightarrow -x^{-2} - y^{-2}\frac{dy}{dx} = 0$

$$\therefore \frac{dy}{dx} = -x^{-2}y^2 = -\left[\frac{y}{x}\right]^2$$

21. $y^2 = x^2 - x \Rightarrow 2y\frac{dy}{dx} = 2x - 1 \Rightarrow \frac{dy}{dx} = \frac{2x - 1}{2y}$

23. $y=\dfrac{\sqrt[3]{x^2+3}}{x}=\dfrac{(x^2+3)^{\frac{1}{3}}}{x} \Rightarrow \dfrac{dy}{dx}=\dfrac{x\left[\frac{1}{3}(x^2+3)^{-\frac{2}{3}}(2x)\right]-(x^2+3)^{\frac{1}{3}}}{x^2}$

$$=\frac{\frac{2}{3}x^2(x^2+3)^{-\frac{2}{3}}-(x^2+3)^{\frac{1}{3}}}{x^2}=\frac{2x^2-3(x^2+3)}{3x^2(x^2+3)^{\frac{2}{3}}}=\frac{-x^2-9}{3x^2(x^2+3)^{\frac{2}{3}}}$$

25. $x^3+y^3=18xy \Rightarrow 3x^2+3y^2\dfrac{dy}{dx}=18\left(x\dfrac{dy}{dx}+y\right)$

$$y^2\frac{dy}{dx}-6x\frac{dy}{dx}=6y-x^2 \Rightarrow \frac{dy}{dx}=\frac{6y-x^2}{y^2-6x}$$

27. $y=(2x+5)^{-\frac{1}{5}} \Rightarrow \dfrac{dy}{dx}=-\dfrac{2}{5}(2x+5)^{-\frac{6}{5}}$

29. $y=\sqrt{1-\sqrt{x}}=(1-x^{\frac{1}{2}})^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{2}(1-x^{\frac{1}{2}})^{-\frac{1}{2}}\dfrac{d}{dx}(1-x^{\frac{1}{2}})$

$$=\frac{1}{2}(1-x^{\frac{1}{2}})^{-\frac{1}{2}}\left(-\frac{1}{2}x^{-\frac{1}{2}}\right)=\frac{-1}{4\sqrt{x}\sqrt{1-\sqrt{x}}}=\frac{-1}{4\sqrt{x-x\sqrt{x}}}$$

31. $y=x^{\frac{1}{2}} \Rightarrow y'=\dfrac{1}{2}x^{-\frac{1}{2}}$; $y(4)=2$ and $y'(4)=\dfrac{1}{4}$.

$\therefore$ Tangent line is: $y-2=\dfrac{1}{4}(x-4)$ or $4y-x=4$.

The x-intercept is $(-4,0)$ and y-intercept is $(0,1)$.

33. $b=a^{\frac{2}{3}} \Rightarrow \dfrac{db}{da}=\dfrac{2}{3}a^{-\frac{1}{3}}$. $\dfrac{db}{da}$ does not exist if $a=0$.

35. $x^2+y^2=1 \Rightarrow 2x+2y\dfrac{dy}{dx}=0 \Rightarrow \dfrac{dy}{dx}=-\dfrac{x}{y}$

$$\frac{d^2y}{dx^2}=-\frac{y-x\frac{dy}{dx}}{y^2}=-\frac{y-x\left(-\frac{x}{y}\right)}{y^2}=-\frac{y^2+x^2}{y^3}=-\frac{1}{y^3}$$

37. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=1 \Rightarrow \dfrac{2}{3}x^{-\frac{1}{3}}+\dfrac{2}{3}y^{-\frac{1}{3}}\dfrac{dy}{dx}=0 \Rightarrow \dfrac{dy}{dx}=-x^{-\frac{1}{3}}y^{\frac{1}{3}}=-\left(\dfrac{y}{x}\right)^{\frac{1}{3}}$

$$\frac{d^2y}{dx^2}=-\left[x^{-\frac{1}{3}}\left(\frac{1}{3}y^{-\frac{2}{3}}\frac{dy}{dx}\right)+y^{\frac{1}{3}}\left(-\frac{1}{3}x^{-\frac{4}{3}}\right)\right]=$$

$$-\frac{1}{3}[(x^{-\frac{1}{3}}y^{-\frac{2}{3}})(-x^{-\frac{1}{3}}y^{\frac{1}{3}})-y^{\frac{1}{3}}x^{-\frac{4}{3}}]=\frac{1}{3}x^{-\frac{4}{3}}y^{-\frac{1}{3}}$$