

In Problems 1–29, find dy/dx .

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|--|----------------------------------|--|-----------------------------------|
| 1. $x^2 + y^2 = 1$ | 2. $y^2 = \frac{x-1}{x+1}$ | 17. $(3x+7)^5 = 2y^3$ | 18. $y = (x+5)^4(x^2-2)^3$ |
| 3. $x^2 - xy = 2$ | 4. $x^2y + xy^2 = 6$ | 19. $\frac{1}{y} + \frac{1}{x} = 1$ | 20. $y = (x^2 + 5x)^3$ |
| 5. $y^2 = x^3$ | 6. $x^{2/3} + y^{2/3} = 1$ | 21. $y^2 = x^2 - x$ | 22. $x^2y^2 = x^2 + y^2$ |
| 7. $x^{1/2} + y^{1/2} = 1$ | 8. $x^3 - xy + y^3 = 1$ | 23. $y = \frac{\sqrt[3]{x^2+3}}{x}$ | |
| 9. $x^2 = \frac{x-y}{x+y}$ | 10. $y = \frac{x}{\sqrt{x^2+1}}$ | 24. $y = x^2\sqrt{1-x^2}$ | |
| 11. $y = x\sqrt{x^2+1}$ | 12. $y^2 = x^2 + \frac{1}{x^2}$ | 25. $x^3 + y^3 = 18xy$ | 26. $y = (3x^2 + 5x + 1)^{3/2}$ |
| 13. $2xy + y^2 = x + y$ | | 27. $y = (2x+5)^{-1/5}$ | 28. $y = 3(2x^{-1/2} + 1)^{-1/3}$ |
| 14. $y = \sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}$ | | 29. $y = \sqrt{1-\sqrt{x}}$ | |
| 15. $y^2 = \frac{x^2-1}{x^2+1}$ | | 30. Find dT/dL if $T^2 = 4\pi^2L/g$. (This equation gives the period T of a simple pendulum in terms of its length L and the acceleration of gravity g .) | |
| 16. $(x+y)^3 + (x-y)^3 = x^4 + y^4$ | | 31. Find the x - and y -intercepts of the line tangent to the curve $y = x^{1/2}$ at $x = 4$. | |

2.3 IMPLICIT DIFFERENTIATION AND FRACTIONAL POWERS

- $x^2 + y^2 = 1 \Rightarrow 2x + 2y \frac{dy}{dx} = 0 \Rightarrow 2y \frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$
- $x^2 - xy = 2 \Rightarrow 2x - x \frac{dy}{dx} - y = 0 \Rightarrow -x \frac{dy}{dx} = y - 2x \Rightarrow$

$$\frac{dy}{dx} = \frac{y-2x}{x}$$
- $y^2 = x^3 \Rightarrow 2y \frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2y}$

$$7. \quad x^{\frac{1}{2}} + y^{\frac{1}{2}} = 1 \Rightarrow \frac{1}{2} x^{-\frac{1}{2}} + \frac{1}{2} y^{-\frac{1}{2}} \frac{dy}{dx} = 0$$

$$y^{-\frac{1}{2}} \frac{dy}{dx} = -x^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = -x^{-\frac{1}{2}} y^{\frac{1}{2}} = -\sqrt{\frac{y}{x}}$$

$$9. \quad x^2 = \frac{x-y}{x+y} \Rightarrow x^3 + x^2 y = x - y \Rightarrow 3x^2 + x^2 \frac{dy}{dx} + 2xy = 1 - \frac{dy}{dx}$$

$$(x^2 + 1) \frac{dy}{dx} = 1 - 2xy - 3x^2 \Rightarrow \frac{dy}{dx} = \frac{1 - 2xy - 3x^2}{x^2 + 1}$$

$$11. \quad y = x\sqrt{x^2+1} = x(x^2+1)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = (x^2+1)^{\frac{1}{2}} + x \left[\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x) \right] = \frac{\sqrt{x^2+1}}{1} + \frac{x^2}{\sqrt{x^2+1}} = \frac{2x^2+1}{\sqrt{x^2+1}}$$

$$13. \quad 2xy + y^2 = x + y \Rightarrow 2x \frac{dy}{dx} + 2y + 2y \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$2x \frac{dy}{dx} + 2y \frac{dy}{dx} - \frac{dy}{dx} = 1 - 2y$$

$$\frac{dy}{dx} = \frac{1 - 2y}{2x + 2y - 1}$$

$$15. \quad y^2 = \frac{x^2 - 1}{x^2 + 1} \Rightarrow 2y \frac{dy}{dx} = \frac{(x^2 + 1)(2x) - (x^2 - 1)(2x)}{(x^2 + 1)^2}$$

$$2y \frac{dy}{dx} = \frac{2x^3 + 2x - 2x^3 + 2x}{(x^2 + 1)^2} = \frac{4x}{(x^2 + 1)^2} \Rightarrow \frac{dy}{dx} = \frac{2x}{y(x^2 + 1)^2}$$

$$17. \quad (3x + 7)^5 = 2y^3 \Rightarrow 5(3x + 7)^4(3) = 6y^2 \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{5(3x + 7)^4}{2y^2}$$

$$19. \quad \frac{1}{x} + \frac{1}{y} = 1 \Rightarrow x^{-1} + y^{-1} = 1 \Rightarrow -x^{-2} - y^{-2} \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -x^{-2} y^2 = -\left[\frac{y}{x}\right]^2$$

$$21. \quad y^2 = x^2 - x \Rightarrow 2y \frac{dy}{dx} = 2x - 1 \Rightarrow \frac{dy}{dx} = \frac{2x - 1}{2y}$$

$$23. \quad y = \frac{\sqrt[3]{x^2+3}}{x} = \frac{(x^2+3)^{\frac{1}{3}}}{x} \Rightarrow \frac{dy}{dx} = \frac{x \left[\frac{1}{3}(x^2+3)^{-\frac{2}{3}}(2x) \right] - (x^2+3)^{\frac{1}{3}}}{x^2}$$

$$= \frac{\frac{2}{3}x^2(x^2+3)^{-\frac{2}{3}} - (x^2+3)^{\frac{1}{3}}}{x^2} = \frac{2x^2 - 3(x^2+3)}{3x^2(x^2+3)^{\frac{2}{3}}} = \frac{-x^2 - 9}{3x^2(x^2+3)^{\frac{2}{3}}}$$

$$25. \quad x^3 + y^3 = 18xy \Rightarrow 3x^2 + 3y^2 \frac{dy}{dx} = 18 \left(x \frac{dy}{dx} + y \right)$$

$$y^2 \frac{dy}{dx} - 6x \frac{dy}{dx} = 6y - x^2 \Rightarrow \frac{dy}{dx} = \frac{6y - x^2}{y^2 - 6x}$$

$$27. \quad y = (2x + 5)^{-\frac{1}{5}} \Rightarrow \frac{dy}{dx} = -\frac{2}{5}(2x + 5)^{-\frac{6}{5}}$$

$$29. \quad y = \sqrt{1 - \sqrt{x}} = (1 - x^{\frac{1}{2}})^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}(1 - x^{\frac{1}{2}})^{-\frac{1}{2}} \frac{d}{dx}(1 - x^{\frac{1}{2}})$$

$$= \frac{1}{2}(1 - x^{\frac{1}{2}})^{-\frac{1}{2}} \left(-\frac{1}{2}x^{-\frac{1}{2}} \right) = \frac{-1}{4\sqrt{x}\sqrt{1 - \sqrt{x}}} = \frac{-1}{4\sqrt{x - x\sqrt{x}}}$$

$$31. \quad y = x^{\frac{1}{2}} \Rightarrow y' = \frac{1}{2}x^{-\frac{1}{2}}; \quad y(4) = 2 \text{ and } y'(4) = \frac{1}{4}.$$

\therefore Tangent line is: $y - 2 = \frac{1}{4}(x - 4)$ or $4y - x = 4$.

The x-intercept is $(-4, 0)$ and y-intercept is $(0, 1)$.

$$33. \quad b = a^{\frac{2}{3}} \Rightarrow \frac{db}{da} = \frac{2}{3}a^{-\frac{1}{3}}. \quad \frac{db}{da} \text{ does not exist if } a = 0.$$

$$35. \quad x^2 + y^2 = 1 \Rightarrow 2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{d^2y}{dx^2} = -\frac{y - x \frac{dy}{dx}}{y^2} = -\frac{y - x \left(-\frac{x}{y} \right)}{y^2} = -\frac{y^2 + x^2}{y^3} = -\frac{1}{y^3}$$

$$37. \quad x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1 \Rightarrow \frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}} \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -x^{\frac{1}{3}} y^{\frac{1}{3}} = -\left(\frac{y}{x} \right)^{\frac{1}{3}}$$

$$\frac{d^2y}{dx^2} = -\left[x^{-\frac{1}{3}} \left(\frac{1}{3}y^{-\frac{2}{3}} \frac{dy}{dx} \right) + y^{\frac{1}{3}} \left(-\frac{1}{3}x^{-\frac{4}{3}} \right) \right] =$$

$$-\frac{1}{3} \left[\left(x^{-\frac{1}{3}} y^{-\frac{2}{3}} \right) \left(-x^{\frac{1}{3}} y^{\frac{1}{3}} \right) - y^{\frac{1}{3}} x^{-\frac{4}{3}} \right] = \frac{1}{3} x^{-\frac{4}{3}} y^{-\frac{1}{3}}$$