TABLE 3.[illegible]

Derivatives	Differentials
1. $\dfrac{d(\sin^{-1}u)}{dx} = \dfrac{du/dx}{\sqrt{1-u^2}}, \quad -1 < u < 1$	1′. $d(\sin^{-1}u) = \dfrac{du}{\sqrt{1-u^2}}, \quad -1 < u < 1$
2. $\dfrac{d(\cos^{-1}u)}{dx} = -\dfrac{du/dx}{\sqrt{1-u^2}}, \quad -1 < u < 1$	2′. $d(\cos^{-1}u) = -\dfrac{du}{\sqrt{1-u^2}}, \quad -1 < u < 1$
3. $\dfrac{d(\tan^{-1}u)}{dx} = \dfrac{du/dx}{1+u^2}$	3′. $d(\tan^{-1}u) = \dfrac{du}{1+u^2}$
4. $\dfrac{d(\cot^{-1}u)}{dx} = -\dfrac{du/dx}{1+u^2}$	4′. $d(\cot^{-1}u) = -\dfrac{du}{1+u^2}$
5. $\dfrac{d(\sec^{-1}u)}{dx} = \dfrac{du/dx}{\lvert u\rvert\sqrt{u^2-1}}, \quad \lvert u\rvert > 1$	5′. $d(\sec^{-1}u) = \dfrac{du}{\lvert u\rvert\sqrt{u^2-1}}, \quad \lvert u\rvert > 1$
6. $\dfrac{d(\csc^{-1}u)}{dx} = \dfrac{-du/dx}{\lvert u\rvert\sqrt{u^2-1}}, \quad \lvert u\rvert > 1$	6′. $d(\csc^{-1}u) = \dfrac{-du}{\lvert u\rvert\sqrt{u^2-1}}, \quad \lvert u\rvert > 1$

PROBLEMS

In Problems 1–20, find dy/dx.

1. $y = \cos^{-1}x^2$

2. $y = \cos^{-1}(1/x)$

3. $y = 5\tan^{-1}3x$

4. $y = \cot^{-1}\sqrt{x}$

5. $y = \sin^{-1}(x/2)$

6. $y = \sin^{-1}(1-x)$

7. $y = \sec^{-1}5x$

8. $y = (1/3)\tan^{-1}(x/3)$

9. $y = \csc^{-1}(x^2+1)$

10. $y = \cos^{-1}2x$

11. $y = \csc^{-1}\sqrt{x} + \sec^{-1}\sqrt{x}$

12. $y = \csc^{-1}\sqrt{x+1}$

13. $y = \cot^{-1}\sqrt{x-1}$

14. $y = x\sqrt{1-x^2} - \cos^{-1}x$

15. $y = \sqrt{x^2-4} - 2\sec^{-1}(x/2)$

16. $y = \cot^{-1}\dfrac{2}{x} + \tan^{-1}\dfrac{x}{2}$

17. $y = \tan^{-1}\dfrac{x-1}{x+1}$

18. $y = x\sin^{-1}x + \sqrt{1-x^2}$

19. $y = x(\sin^{-1}x)^2 - 2x + 2\sqrt{1-x^2}\sin^{-1}x$

20. $y = x\cos^{-1}2x - (1/2)\sqrt{1-4x^2}$

Evaluate the integrals in Problems 21–40.

21. $\displaystyle\int_{[illegible]}^{1/2} \frac{dx}{\sqrt{1-x^2}}$

22. $\displaystyle\int_{-1}^{1} \frac{dx}{1+x^2}$

23. $\displaystyle\int_{[illegible]}^{[illegible]} \frac{dx}{x\sqrt{x^2-1}}$

24. $\displaystyle\int_{-2}^{-\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}}$

25. $\displaystyle\int_{[illegible]}^{[illegible]} \frac{4\,dx}{1+x^2}$

26. $\displaystyle\int_{\sqrt{3}/3}^{\sqrt{3}} \frac{6\,dx}{1+x^2}$

27. $\displaystyle\int_{0}^{\sqrt{2}/2} \frac{x\,dx}{\sqrt{1-x^4}}$

28. $\displaystyle\int_{0}^{1/4} \frac{dx}{\sqrt{1-4x^2}}$

29. $\displaystyle\int_{1/\sqrt{3}}^{1} \frac{dx}{x\sqrt{4x^2-1}}$

30. $\displaystyle\int_{0}^{1} \frac{x}{1+x^4}\,dx$

31. $\displaystyle\int_{0}^{\sqrt{2}} \frac{4x\,dx}{\sqrt{4-x^4}}$

32. $\displaystyle\int_{0}^{1} \frac{dx}{\sqrt{4-x^2}}$

33. $\displaystyle\int_{\sqrt{2}}^{\sqrt[4]{2}} \frac{x\,dx}{x^2\sqrt{x^4-1}}$

34. $\displaystyle\int_{2}^{4} \frac{dx}{2x\sqrt{x-1}}$ (*Hint*: Let $x = u^2$.)

35. $\displaystyle\int_{0}^{2} \frac{dx}{1+(x-1)^2}$

36. $\displaystyle\int_{1}^{3} \frac{2\,dx}{\sqrt{x}(1+x)}$

37. $\displaystyle\int_{1/2}^{3/4} \frac{dx}{\sqrt{x}\sqrt{1-x}}$

38. $\displaystyle\int_{3/2}^{1+\sqrt{2}/2} \frac{dx}{\sqrt{1-(x-1)^2}}$

39. $\displaystyle\int_{-2/3}^{-\sqrt{2}/3} \frac{dx}{x\sqrt{9x^2-1}}$

40. $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{2\cos x\,dx}{1+\sin^2 x}$

Use l'Hôpital's rule (Article 3.8) to evaluate the limits in Problems 41–44.

41. $\displaystyle\lim_{x\to 0} \frac{\sin^{-1}2x}{x}$

42. $\displaystyle\lim_{x\to 0} \frac{2\tan^{-1}3x}{5x}$

6.3 DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

1. $y = \cos^{-1} x^2 \Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-(x^2)^2}} \frac{d}{dx}(x^2) = \frac{-2x}{\sqrt{1-x^4}}$.

3. $y = 5\tan^{-1} 3x \Rightarrow \frac{dy}{dx} = 5\left(\frac{1}{1+(3x)^2}\right)\frac{d}{dx}(3x) = \frac{15}{1+9x^2}$.

5. $y = \sin^{-1} \frac{x}{2} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{\sqrt{4-x^2}}$.

7. $y = \sec^{-1} 5x \Rightarrow \frac{dy}{dx} = \frac{1}{|5x|\sqrt{(5x)^2-1}} \frac{d}{dx}(5x) = \frac{1}{|x|\sqrt{25x^2-1}}$.

9. $y = \csc^{-1}(x^2+1) \Rightarrow \frac{dy}{dx} = \frac{-1}{|x^2+1|\sqrt{(x^2+1)^2-1}}\frac{d}{dx}(x^2+1) = \frac{-2x}{(x^2+1)\sqrt{x^4+2x^2}}$.

11. $y = \csc^{-1}\sqrt{x} + \sec^{-1}\sqrt{x} = \frac{\pi}{2}$ since the secant and cosecant are cofunctions. $\therefore \frac{dy}{dx} = 0$.

13. $y = \cot^{-1}\sqrt{x-1} \Rightarrow \frac{dy}{dx} = \frac{-1}{1+(\sqrt{x-1})^2}\frac{d}{dx}(\sqrt{x-1}) = \frac{-1}{2x\sqrt{x-1}}$.

15. $y = \sqrt{x^2-4} - 2\sec^{-1}\frac{x}{2} \Rightarrow \frac{dy}{dx} = \frac{1}{2}(x^2-4)^{-\frac{1}{2}}(2x) - 2\left(\frac{\frac{1}{2}}{|\frac{x}{2}|\sqrt{\left(\frac{x}{2}\right)^2-1}}\right)$

$= \frac{x}{\sqrt{x^2-4}} - \frac{1}{|\frac{x}{2}|\sqrt{\frac{x^2-4}{4}}} = \frac{x|x|-4}{|x|\sqrt{x^2-4}}$.

If $x > 2$, $|x| = x$ and $\frac{dy}{dx} = \frac{\sqrt{x^2-4}}{x}$. If $x < -2$, $|x| = -x$ and $\frac{dy}{dx} = -\frac{x^2+4}{x\sqrt{x^2-4}}$.

17. $y = \tan^{-1}\frac{x-1}{x+1} \Rightarrow \frac{dy}{dx} = \frac{1}{1+\left(\frac{x-1}{x+1}\right)^2}\frac{d}{dx}\left(\frac{x-1}{x+1}\right)$

$= \left(\frac{(x+1)^2}{(x+1)^2+(x-1)^2}\right)\left(\frac{x+1-x+1}{(x+1)^2}\right) = \frac{1}{x^2+1}$.

19. $y = x(\sin^{-1}x)^2 - 2x + 2\sqrt{1-x^2}\sin^{-1}x$

$\frac{dy}{dx} = (\sin^{-1}x)^2 + x(2\sin^{-1}x)\left(\frac{1}{\sqrt{1-x^2}}\right) - 2 + 2\left[\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)\sin^{-1}x\right.$

$\left. + 2\sqrt{1-x^2}\left(\frac{1}{\sqrt{1-x^2}}\right)\right] =$

$(\sin^{-1}x)^2 + \frac{2x\sin^{-1}x}{\sqrt{1-x^2}} + \frac{-2x\sin^{-1}x}{\sqrt{1-x^2}} = (\sin^{-1}x)^2$.

21. $\displaystyle\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x \Big]_0^{\frac{1}{2}} = \sin^{-1}\frac{1}{2} - \sin^{-1}0 = \frac{\pi}{6}$

23. $\displaystyle\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}|x| \Big]_{\sqrt{2}}^{2} = \sec^{-1}2 - \sec^{-1}\sqrt{2} = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

25. $\displaystyle\int_{-1}^{0} \frac{4dx}{1+x^2} = 4\tan^{-1}x \Big]_{-1}^{0} = 0 - 4\tan^{-1}(-1) = \pi$

27. $\displaystyle\int_0^{\frac{\sqrt{2}}{2}} \frac{xdx}{\sqrt{1-x^4}} = \int_0^{\frac{1}{2}} \frac{\frac{1}{2}du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1}u \Big]_0^{\frac{1}{2}} = \frac{\pi}{12}$

Let $u = x^2 \Rightarrow du = 2xdx$. Then $x = \frac{\sqrt{2}}{2} \Rightarrow u = \frac{1}{2}$; $x = 0 \Rightarrow u = 0$.

29. $\displaystyle\int_{\frac{1}{\sqrt{3}}}^{1} \frac{dx}{x\sqrt{4x^2-1}} = \int_{\frac{2}{\sqrt{3}}}^{2} \frac{\frac{1}{2}du}{\frac{1}{2}u\sqrt{u^2-1}} = \sec^{-1}u \Big]_{\frac{2}{\sqrt{3}}}^{2} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

Let $u = 2x \Rightarrow u^2 = 4x^2$ and $du = 2dx$. $x = \frac{1}{\sqrt{3}} \Rightarrow u = \frac{2}{\sqrt{3}}$ and $x = 1 \Rightarrow u = 2$

31. $\displaystyle\int_0^{\sqrt{2}} \frac{4x\ dx}{\sqrt{4-x^4}} = 2\int_0^{\frac{\pi}{2}} du = 2u\Big]_0^{\frac{\pi}{2}} = \pi$. Let $x^2 = 2\sin u \Rightarrow 2x\ dx$

$= 2\cos u\ du$. $x = 0 \Rightarrow u = 0$; $x = \sqrt{2} \Rightarrow u = \frac{\pi}{2}$.

33. $\displaystyle\int_{\sqrt{2}}^{\sqrt[4]{2}} \frac{xdx}{x^2\sqrt{x^4-1}} = \int_2^{\sqrt{2}} \frac{\frac{1}{2}du}{u\sqrt{u^2-1}} = \frac{1}{2}\sec^{-1}|u| \Big]_2^{\sqrt{2}} = -\frac{\pi}{24}$

Let $u = x^2 \Rightarrow du = 2xdx$; $x = \sqrt{2} \Rightarrow u = 2$ and $x = \sqrt[4]{2} \Rightarrow u = \sqrt{2}$.

35. $\displaystyle\int_0^2 \frac{dx}{1+(x-1)^2} = \tan^{-1}(x-1)\Big]_0^2 = \tan^{-1}1 - \tan^{-1}(-1) = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

37. $\displaystyle\int_{\frac{1}{2}}^{\frac{3}{4}} \frac{dx}{\sqrt{x}\sqrt{1-x}} = \int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{2udu}{u\sqrt{1-u^2}} = 2\sin^{-1}u\Big]_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} = \frac{\pi}{6}$

Let $u = \sqrt{x} \Rightarrow u^2 = x$ and $2udu = dx$. $x = \frac{1}{2} \Rightarrow u = \frac{1}{\sqrt{2}}$ and $x = \frac{3}{4} \Rightarrow u = \frac{\sqrt{3}}{2}$

39. $\displaystyle\int_{-\frac{2}{3}}^{-\frac{\sqrt{2}}{3}} \frac{dx}{x\sqrt{9x^2-1}} = \int_{-2}^{-\sqrt{2}} \frac{\frac{1}{3}du}{\frac{1}{3}u\sqrt{u^2-1}} = \sec^{-1}|u|\Big]_{-2}^{-\sqrt{2}} = -\frac{\pi}{12}$

Let $u = 3x \Rightarrow u^2 = 9x^2$ and $du = 3dx$. $x = -\frac{2}{3} \Rightarrow u = -2$; $x = -\frac{\sqrt{2}}{3} \Rightarrow u = -\sqrt{2}$

41. $\displaystyle\lim_{x\to 0} \frac{\sin^{-1}2x}{x} = \lim_{x\to 0} \frac{\frac{2}{\sqrt{1-4x^2}}}{1} = 2$

43. $\displaystyle\lim_{x\to 0} x^{-3}(\sin^{-1}x - x) = \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}} - 1}{3x^2} = \lim_{x\to 0} \frac{\frac{x}{\sqrt{1-x^2}}}{6x} = \frac{1}{6}$

45. $f(x) = \sin^{-1}x + \cos^{-1}x \equiv \frac{\pi}{2}$. $\therefore f'(x) = 0$ and $f(.32) = \frac{\pi}{2}$.

47. $\tan\theta = x \Rightarrow \sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}$. $x = 2 \Rightarrow \sec\theta = \sqrt{5}$.

$(\sqrt{5})^2 \frac{d\theta}{dt} = 4 \Rightarrow \frac{d\theta}{dt} = \frac{4}{5}$ rad/hr

49. Yes, $\sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x \Rightarrow$ these functions differ by at most a constant.