Find the limits in Problems 1–23.

1. $\lim_{x\to 2}\dfrac{x-2}{x^2-4}$

2. $\lim_{t\to\infty}\dfrac{6t+5}{3t-8}$

3. $\lim_{x\to\infty}\dfrac{5x^2-3x}{7x^2+1}$

4. $\lim_{x\to 1}\dfrac{x^3-1}{4x^3-x-3}$

5. $\lim_{t\to 0}\dfrac{\sin t^2}{t}$

6. $\lim_{x\to\pi/2}\dfrac{2x-\pi}{\cos x}$

7. $\lim_{x\to 0}\dfrac{\sin 5x}{x}$

8. $\lim_{t\to 0}\dfrac{\cos t-1}{t^2}$

9. $\lim_{\theta\to\pi}\dfrac{\sin\theta}{\pi-\theta}$

10. $\lim_{x\to\pi/2}\dfrac{1-\sin x}{1+\cos 2x}$

11. $\lim_{x\to\pi/4}\dfrac{\sin x-\cos x}{x-\pi/4}$

12. $\lim_{x\to\pi/3}\dfrac{\cos x-0.5}{x-\pi/3}$

13. $\lim_{x\to(\pi/2)^-}\left(x-\dfrac{\pi}{2}\right)\tan x$

14. $\lim_{x\to 0}\dfrac{2x}{x+7\sqrt{x}}$

15. $\lim_{x\to 1}\dfrac{2x^2-(3x+1)\sqrt{x}+2}{x-1}$

16. $\lim_{x\to 2}\dfrac{\sqrt{x^2+5}-3}{x^2-4}$

17. $\lim_{x\to 0}\dfrac{\sqrt{a(a+x)}-a}{x},\quad a>0$

18. $\lim_{t\to 0}\dfrac{10\,(\sin t-t)}{t^3}$

19. $\lim_{x\to 0}\dfrac{x(\cos x-1)}{\sin x-x}$

20. $\lim_{h\to 0}\dfrac{\sin(a+h)-\sin a}{h}$

21. $\lim_{r\to 1}\dfrac{a(r^n-1)}{r-1}$, n a positive integer

22. $\lim_{x\to 0^+}\left(\dfrac{1}{x}-\dfrac{1}{\sqrt{x}}\right)$

23. $\lim_{x\to\infty}(x-\sqrt{x^2+x})$

24. Which is correct, (a) or (b)? Explain.

a) $\lim_{x\to 3}\dfrac{x-3}{x^2-3}=\lim_{x\to 3}\dfrac{1}{2x}=\dfrac{1}{6}$ b) $\lim_{x\to 3}\dfrac{x-3}{x^2-3}=\dfrac{0}{6}=0$

25. L'Hôpital's rule does not help with $\lim_{x\to\infty}\dfrac{\sqrt{10x+1}}{\sqrt{x+1}}$. Find the limit some other way.

26. L'Hôpital's rule does not work with

$$\lim_{x\to\pi/2}\frac{\sec x}{\tan x}.$$

Try it—you just keep on going. Find the limit some other way.

27. Let $y=\sec x+\tan x$, $-\pi/2<x<\pi/2$.

a) Show that y, y', and y'' are positive.

b) Find

$$\lim_{x\to-(\pi/2)^+}(\sec x+\tan x)$$

c) Graph y for $-\pi/2<x<\pi/2$.

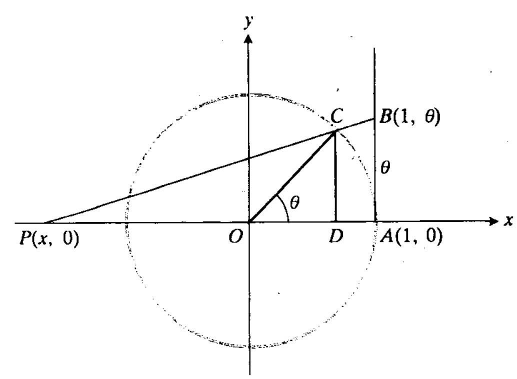
28. In Fig. 3.68, the circle has radius OA equal to 1, and AB is tangent to the circle at A. The arc AC has radian measure θ and the segment AB also has length θ. The line through B and C crosses the x-axis at $P(x, 0)$.

a) Show that the length of PA is

$$1-x=\frac{\theta(1-\cos\theta)}{\theta-\sin\theta}.$$

b) Find $\lim_{\theta\to 0}(1-x)$.

c) Show that $\lim_{\theta\to\infty}[(1-x)-(1-\cos\theta)]=0$.



3.68 The diagram for Problem 28.

3.8 INDETERMINATE FORMS AND L'HOPITAL'S RULE

1. $\lim_{x\to 2}\frac{x-2}{x^2-4}=\lim_{x\to 2}\frac{1}{2x}=\frac{1}{4}$

3. $\lim_{x\to\infty}\frac{5x^2-3x}{7x^2+1}=\lim_{x\to\infty}\frac{10x-3}{14x}=\lim_{x\to\infty}\frac{10}{14}=\frac{5}{7}$

5. $\lim_{t\to 0}\frac{\sin t^2}{t}=\lim_{t\to 0}\frac{2\sin t\cos t}{1}=0$

7. $\lim_{x\to 0}\frac{\sin 5x}{x}=\lim_{x\to 0}\frac{5\cos 5x}{1}=5$

9. $\lim_{\theta\to\pi}\frac{\sin\theta}{\pi-\theta}=\lim_{\theta\to\pi}\frac{\cos\theta}{-1}=\frac{-1}{-1}=1$

11. $\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{x-\frac{\pi}{4}}=\lim_{x\to\frac{\pi}{4}}\frac{\cos x+\sin x}{1}=\sqrt{2}$

13. $\lim_{x\to\frac{\pi}{2}}\left[-\left(x-\frac{\pi}{2}\right)\tan x\right]=\lim_{x\to\frac{\pi}{2}}\left[\frac{-\left(x-\frac{\pi}{2}\right)}{\cot x}\right]=\lim_{x\to\frac{\pi}{2}}\left(\frac{-1}{-\csc^2 x}\right)=1$

15. $\lim_{x\to 1}\frac{2x^2-(3x+1)\sqrt{x}+2}{x-1}=\lim_{x\to 1}\frac{4x-3\sqrt{x}-\frac{3x+1}{2\sqrt{x}}}{1}=-1$

17. $\lim_{x\to 0}\frac{\sqrt{a(a+x)}-a}{x}=\lim_{x\to 0}\frac{\frac{1}{2}[a(a+x)]^{-\frac{1}{2}}(a)}{1}=\frac{1}{2}$

19. $\lim_{x\to 0}\frac{x(\cos x-1)}{\sin x-x}=\lim_{x\to 0}\frac{-x\sin x+\cos x-1}{\cos x-1}=$

$\lim_{x\to 0}\frac{-x\cos x-\sin x-\sin x}{-\sin x}=\lim_{x\to 0}\frac{x\sin x-\cos x-2\cos x}{-\cos x}=3$

21. $\lim_{r\to 1}\frac{a(r^n-1)}{r-1}$ (n a positive integer) $=\lim_{r\to 1}\frac{nar^{n-1}}{1}=na$

23. $\lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right)=\lim_{x\to\infty}\frac{x-\sqrt{x^2+x}}{1}\cdot\frac{x+\sqrt{x^2+x}}{x+\sqrt{x^2+x}}$

$\lim_{x\to\infty}\frac{-x}{x+\sqrt{x^2+x}}=\lim_{x\to\infty}\frac{-\frac{x}{x}}{\frac{x}{x}+\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}}=\lim_{x\to\infty}\frac{-1}{1+\sqrt{1+\frac{1}{x}}}=-\frac{1}{2}$

25. $\lim_{x\to\infty}\frac{\sqrt{10x+1}}{\sqrt{x+1}}=\lim_{x\to\infty}\frac{\sqrt{10+\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}=\sqrt{10}$

27. (a) $y=\sec x+\tan x,\ -\frac{\pi}{2}<x<\frac{\pi}{2}.\ \ y=\frac{1+\sin x}{\cos x}\geq 0$

since $|\sin x|\leq 1 \Rightarrow 1+\sin x>0$, and $\cos x>0$ for $-\frac{\pi}{2}<x<\frac{\pi}{2}$.

$\therefore\ y'=\sec x\tan x+\sec^2 x=\frac{\sin x+1}{\cos^2 x}\geq 0$ also.

$y''=\sec x(\sec^2 x)+\tan x(\sec x\tan x)+2\sec x(\sec x\tan x)$

$=\frac{1}{\cos^3 x}+\frac{\sin^2 x}{\cos^3 x}+\frac{2\sin x}{\cos^3 x}=\frac{(1+\sin x)^2}{\cos^3 x}\geq 0.$

(b) $\lim_{x\to\left(-\frac{\pi}{2}\right)^+}(\sec x+\tan x)=\lim_{x\to\left(-\frac{\pi}{2}\right)^+}\frac{1+\sin x}{\cos x}=\lim_{x\to\left(-\frac{\pi}{2}\right)^+}\frac{\cos x}{-\sin x}=0$

(c)