

Find the limits in Problems 1–23.

1.  $\lim_{x \rightarrow 2} \frac{x-2}{x^2-4}$
  2.  $\lim_{t \rightarrow \infty} \frac{6t+5}{3t-8}$
  3.  $\lim_{x \rightarrow \infty} \frac{5x^2-3x}{7x^2+1}$
  4.  $\lim_{x \rightarrow 1} \frac{x^3-1}{4x^3-x-3}$
  5.  $\lim_{t \rightarrow 0} \frac{\sin t^2}{t}$
  6.  $\lim_{x \rightarrow \pi/2} \frac{2x-\pi}{\cos x}$
  7.  $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$
  8.  $\lim_{t \rightarrow 0} \frac{\cos t-1}{t^2}$
  9.  $\lim_{\theta \rightarrow \pi} \frac{\sin \theta}{\pi-\theta}$
  10.  $\lim_{x \rightarrow \pi/2} \frac{1-\sin x}{1+\cos 2x}$
  11.  $\lim_{x \rightarrow \pi/4} \frac{\sin x - \cos x}{x - \pi/4}$
  12.  $\lim_{x \rightarrow \pi/3} \frac{\cos x - 0.5}{x - \pi/3}$
  13.  $\lim_{x \rightarrow (\pi/2)^-} -\left(x - \frac{\pi}{2}\right) \tan x$
  14.  $\lim_{x \rightarrow 0} \frac{2x}{x + 7\sqrt{x}}$
  15.  $\lim_{x \rightarrow 1} \frac{2x^2 - (3x+1)\sqrt{x} + 2}{x-1}$
  16.  $\lim_{x \rightarrow 2} \frac{\sqrt{x^2+5}-3}{x^2-4}$
  17.  $\lim_{x \rightarrow 0} \frac{\sqrt{a(a+x)} - a}{x}, \quad a > 0$
  18.  $\lim_{t \rightarrow 0} \frac{10(\sin t - t)}{t^3}$
  19.  $\lim_{x \rightarrow 0} \frac{x(\cos x - 1)}{\sin x - x}$
  20.  $\lim_{h \rightarrow 0} \frac{\sin(a+h) - \sin a}{h}$
  21.  $\lim_{r \rightarrow 1} \frac{a(r^n - 1)}{r - 1}, \quad n \text{ a positive integer}$
  22.  $\lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{\sqrt{x}} \right)$
  23.  $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x})$
24. Which is correct, (a) or (b)? Explain.
- a)  $\lim_{x \rightarrow 3} \frac{x-3}{x^2-3} = \lim_{x \rightarrow 3} \frac{1}{2x} = \frac{1}{6}$       b)  $\lim_{x \rightarrow 3} \frac{x-3}{x^2-3} = \frac{0}{6} = 0$
25. L'Hôpital's rule does not help with  $\lim_{x \rightarrow \infty} \frac{\sqrt{10x+1}}{\sqrt{x+1}}$ . Find the limit some other way.

26. L'Hôpital's rule does not work with

$$\lim_{x \rightarrow \pi/2} \frac{\sec x}{\tan x}$$

Try it—you just keep on going. Find the limit some other way.

27. Let  $y = \sec x + \tan x, -\pi/2 < x < \pi/2$ .

- a) Show that  $y, y',$  and  $y''$  are positive.
- b) Find

$$\lim_{x \rightarrow -(\pi/2)^+} (\sec x + \tan x)$$

c) Graph  $y$  for  $-\pi/2 < x < \pi/2$ .

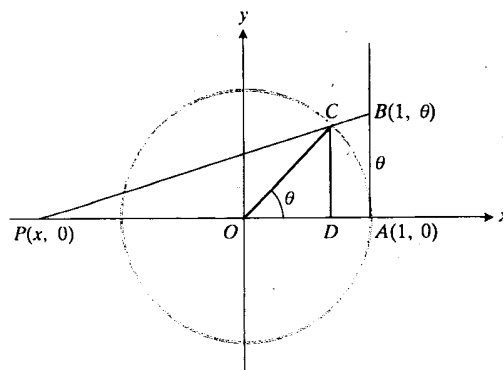
28. In Fig. 3.68, the circle has radius  $OA$  equal to 1, and  $AB$  is tangent to the circle at  $A$ . The arc  $AC$  has radian measure  $\theta$  and the segment  $AB$  also has length  $\theta$ . The line through  $B$  and  $C$  crosses the  $x$ -axis at  $P(x, 0)$ .

a) Show that the length of  $PA$  is

$$1-x = \frac{\theta(1-\cos \theta)}{\theta - \sin \theta}$$

b) Find  $\lim_{\theta \rightarrow 0} (1-x)$ .

c) Show that  $\lim_{\theta \rightarrow \infty} [(1-x) - (1-\cos \theta)] = 0$ .



3.68 The diagram for Problem 28.

### 3.8 INDETERMINATE FORMS AND L'HOPITAL'S RULE

$$1. \lim_{x \rightarrow 2} \frac{x-2}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{1}{2x} = \frac{1}{4}$$

$$3. \lim_{x \rightarrow \infty} \frac{5x^2 - 3x}{7x^2 + 1} = \lim_{x \rightarrow \infty} \frac{10x - 3}{14x} = \lim_{x \rightarrow \infty} \frac{10}{14} = \frac{5}{7}$$

$$5. \lim_{t \rightarrow 0} \frac{\sin t^2}{t} = \lim_{t \rightarrow 0} \frac{2 \sin t \cos t}{1} = 0$$

$$7. \lim_{x \rightarrow 0} \frac{\sin 5x}{x} = \lim_{x \rightarrow 0} \frac{5 \cos 5x}{1} = 5$$

$$9. \lim_{\theta \rightarrow \pi} \frac{\sin \theta}{\pi - \theta} = \lim_{\theta \rightarrow \pi} \frac{\cos \theta}{-1} = \frac{-1}{-1} = 1$$

$$11. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x + \sin x}{1} = \sqrt{2}$$

$$13. \lim_{x \rightarrow \frac{\pi}{2}} \left[ -\left(x - \frac{\pi}{2}\right) \tan x \right] = \lim_{x \rightarrow \frac{\pi}{2}} \left[ \frac{-\left(x - \frac{\pi}{2}\right)}{\cot x} \right] = \lim_{x \rightarrow \frac{\pi}{2}} \left( \frac{-1}{-\csc x} \right) = 1$$

$$15. \lim_{x \rightarrow 1} \frac{2x^2 - (3x+1)\sqrt{x} + 2}{x-1} = \lim_{x \rightarrow 1} \frac{4x - 3\sqrt{x} - \frac{3x+1}{2\sqrt{x}}}{1} = -1$$

$$17. \lim_{x \rightarrow 0} \frac{\sqrt{a(a+x)} - a}{x} = \lim_{x \rightarrow 0} \frac{\frac{1}{2}[a(a+x)]^{-\frac{1}{2}}(a)}{1} = \frac{1}{2}$$

$$19. \lim_{x \rightarrow 0} \frac{x(\cos x - 1)}{\sin x - x} = \lim_{x \rightarrow 0} \frac{-x \sin x + \cos x - 1}{\cos x - 1} =$$

$$\lim_{x \rightarrow 0} \frac{-x \cos x - \sin x - \sin x}{-\sin x} = \lim_{x \rightarrow 0} \frac{x \sin x - \cos x - 2 \cos x}{-\cos x} = 3$$

$$21. \lim_{r \rightarrow 1} \frac{a(r^n - 1)}{r - 1} \text{ (n a positive integer)} = \lim_{r \rightarrow 1} \frac{nar^{n-1}}{1} = na$$

$$23. \lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x}) = \lim_{x \rightarrow \infty} \frac{x - \sqrt{x^2 + x}}{1} \cdot \frac{x + \sqrt{x^2 + x}}{x + \sqrt{x^2 + x}}$$

$$\lim_{x \rightarrow \infty} \frac{-x}{x + \sqrt{x^2 + x}} = \lim_{x \rightarrow \infty} \frac{\frac{-x}{x}}{\frac{x}{x} + \sqrt{\frac{x^2}{x^2} + \frac{x}{x^2}}} = \lim_{x \rightarrow \infty} \frac{-1}{1 + \sqrt{1 + \frac{1}{x}}} = -\frac{1}{2}$$

$$25. \lim_{x \rightarrow \infty} \frac{\sqrt{10x+1}}{\sqrt{x+1}} = \lim_{x \rightarrow \infty} \frac{\sqrt{10 + \frac{1}{x}}}{\sqrt{1 + \frac{1}{x}}} = \sqrt{10}$$

$$27. (a) y = \sec x + \tan x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}. \quad y = \frac{1 + \sin x}{\cos x} \geq 0$$

since  $|\sin x| \leq 1 \Rightarrow 1 + \sin x > 0$ , and  $\cos x > 0$  for  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ .

$$\therefore y' = \sec x \tan x + \sec^2 x = \frac{\sin x + 1}{\cos^2 x} \geq 0 \text{ also.}$$

$$y'' = \sec x (\sec^2 x) + \tan x (\sec x \tan x) + 2 \sec x (\sec x \tan x)$$

$$= \frac{1}{\cos^3 x} + \frac{\sin^2 x}{\cos^3 x} + \frac{2 \sin x}{\cos^3 x} = \frac{(1 + \sin x)^2}{\cos^3 x} \geq 0.$$

$$(b) \lim_{x \rightarrow (-\frac{\pi}{2})^+} (\sec x + \tan x) = \lim_{x \rightarrow (-\frac{\pi}{2})^+} \frac{1 + \sin x}{\cos x} = \lim_{x \rightarrow (-\frac{\pi}{2})^+} \frac{\cos x}{-\sin x} = 0$$

