

Important Formulas

1. $\ln x = \int_1^x \frac{1}{t} dt, \quad x > 0$ (definition of $\ln x$)
2. $\frac{d}{dx} \ln x = \frac{1}{x}$
3. $\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$
4. $\int \frac{du}{u} = \begin{cases} \ln u + C & \text{if } u > 0, \\ \ln(-u) + C' & \text{if } u < 0 \end{cases}$
5. $\int \frac{du}{u} = \ln|u| + C$ (if u does not change sign on the domain of integration)

PROBLEMS

In Problems 1–20, find dy/dx .

1. $y = \ln 2x$
2. $y = \ln 5x$
3. $y = \ln kx$ (k constant)
4. $y = (\ln x)^2$
5. $y = \ln(10/x)$
6. $y = \ln(x^2 + 2x)$
7. $y = (\ln x)^3$
8. $y = \ln(\cos x)$
9. $y = \ln(\sec x + \tan x)$
10. $y = x \ln x - x$
11. $y = x^3 \ln(2x)$
12. $y = \ln(\csc x)$
13. $y = \tan^{-1}(\ln x)$
14. $y = \ln(\ln x)$
15. $y = x^2 \ln(x^2)$
16. $y = \ln(x^2 + 4) - x \tan^{-1} \frac{x}{2}$
17. $y = \ln x - \frac{1}{2} \ln(1 + x^2) - \frac{\tan^{-1} x}{x}$
18. $y = x(\ln x)^3$
19. $y = x[\sin(\ln x) + \cos(\ln x)]$
20. $y = x \ln(a^2 + x^2) - 2x + 2a \tan^{-1} \frac{x}{a}$

Evaluate the integrals in Problems 21–40.

21. $\int \frac{dx}{x}$
22. $\int \frac{2 dx}{x}$
23. $\int \frac{dx}{2x}$
24. $\int \frac{dx}{x+2}$
25. $\int_0^1 \frac{dx}{x+1}$
26. $\int_{-1}^0 \frac{dx}{1-x}$
27. $\int_{-1}^0 \frac{dx}{2x+3}$
28. $\int_{-1}^0 \frac{3 dx}{2-3x}$
29. $\int_0^1 \frac{x dx}{4x^2+1}$
30. $\int_0^\pi \frac{\sin x dx}{2-\cos x}$

31. $\int \tan 3x dx$
32. $\int \cot 5x dx$
33. $\int \frac{x^2 dx}{4-x^3}$
34. $\int \frac{\sec^2 2x dx}{1+\tan 2x}$
35. $\int \frac{dx}{x \ln x}$
36. $\int \frac{dx}{x(\ln x)^2}$
37. $\int_1^2 \frac{(\ln x)^2 dx}{x}$
38. $\int_1^3 \frac{\cos(\ln x) dx}{x}$
39. $\int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$
40. $\int \frac{dx}{(1+x^2) \tan^{-1} x}$

Evaluate the limits in Problems 41–44.

41. $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$
42. $\lim_{x \rightarrow \infty} \frac{\ln(\ln x)}{\ln x}$
43. $\lim_{t \rightarrow 0} \frac{\ln(1+2t) - 2t}{t^2}$
44. $\lim_{\theta \rightarrow 0^+} \frac{\ln(\sin \theta)}{\cot \theta}$
45. Find the area of the “triangular” region in the first quadrant bounded by the lines $x = 1$ and $y = 1$ and the hyperbola $xy = 2$.

and hence $\ln x$ increases without limit as x does. That is,

$$\ln x \rightarrow +\infty \quad \text{as} \quad x \rightarrow +\infty. \quad (4)$$

On the other hand, as x approaches zero through positive values, $1/x$ tends to plus infinity. Hence, from Eq. (4) we have

$$\ln x = -\ln \frac{1}{x} \rightarrow -\infty \quad \text{as} \quad x \rightarrow 0^+. \quad (5)$$

The y -axis is a vertical asymptote of the graph of $y = \ln x$.

Properties of $y = \ln x$

1. Domain: The set of all positive real numbers, $x > 0$.
2. Range: The set of all real numbers $-\infty < y < \infty$.
3. It is a continuous, increasing function everywhere on its domain. If $x_1 > x_2 > 0$, then $\ln x_1 > \ln x_2$. It is a one-to-one function from its domain to its range. (It therefore has an inverse, which will be the subject of the next article.)
4. Products, quotients, and powers: If a and x are any two positive numbers, then

$$\ln ax = \ln a + \ln x, \quad (6)$$

$$\ln \frac{x}{a} = \ln x - \ln a, \quad (7)$$

$$\ln x^n = n \ln x. \quad (8)$$

PROBLEMS

Express the logarithms in Problems 1–10 in terms of $\ln 2$ and $\ln 3$. For example, $\ln 1.5 = \ln(3/2) = \ln 3 - \ln 2$.

1. $\ln 16$
2. $\ln \sqrt[3]{9}$
3. $\ln 2\sqrt{2}$
4. $\ln 0.25$
5. $\ln 4/9$
6. $\ln 12$
7. $\ln 9/8$
8. $\ln 36$
9. $\ln 4.5$
10. $\ln \sqrt{13.5}$

In Problems 11–22, find dy/dx .

11. $y = \ln \sqrt{x^2 + 5}$
12. $y = \ln x^{3/2}$
13. $y = \ln \frac{1}{x\sqrt{x+1}}$
14. $y = \ln \sqrt[3]{\cos x}$
15. $y = \ln(\sin x \sin 2x)$
16. $y = \ln(x\sqrt{x^2 + 1})$
17. $y = \ln(3x\sqrt{x+2})$
18. $y = \frac{1}{2} \ln \frac{1+x}{1-x}$
19. $y = \frac{1}{3} \ln \frac{x^3}{1+x^3}$
20. $y = \ln \frac{x}{2+3x}$
21. $y = \ln \frac{(x^2+1)^5}{\sqrt{1-x}}$
22. $y = \int_{\sqrt{x}}^{\sqrt[3]{x}} \ln t \, dt$

In Problems 23–31, find dy/dx by logarithmic differentiation.

23. $y^2 = x(x+1), \quad x > 0$
24. $y = \sqrt[3]{\frac{x+1}{x-1}}, \quad x > 1$
25. $y = \sqrt{x+3} \sin x \cos x, \quad 0 < x < \pi/2$
26. $y = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}, \quad x > 0$
27. $y = \sqrt[3]{\frac{x(x-2)}{x^2+1}}, \quad x > 2$
28. $y^5 = \sqrt{\frac{(x+1)^5}{(x+2)^{10}}}$
29. $y = \sqrt[3]{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}, \quad x > 2$
30. $y^{4/5} = \frac{\sqrt{\sin x \cos x}}{1+2 \ln x}$
31. $\sqrt{y} = \frac{x^5 \tan^{-1} x}{(3-2x)\sqrt[3]{x}}$

6.4 THE NATURAL LOGARITHM AND ITS DERIVATIVE

$$1. \quad y = \ln 2x \Rightarrow \frac{dy}{dx} = \frac{1}{2x} \frac{d}{dx}(2x) = \frac{2}{2x} = \frac{1}{x}$$

$$3. \quad y = \ln kx \Rightarrow \frac{dy}{dx} = \frac{1}{kx} \frac{d}{dx}(kx) = \frac{k}{kx} = \frac{1}{x}$$

$$5. \quad y = \ln\left(\frac{10}{x}\right) = \ln(10x^{-1}) = -\ln 10x \Rightarrow \frac{dy}{dx} = -\frac{1}{x}$$

$$7. \quad y = (\ln x)^3 \Rightarrow \frac{dy}{dx} = 3(\ln x)^2 \frac{1}{x} = \frac{3\ln^2 x}{x}$$

$$9. \quad y = \ln(\sec x + \tan x) \Rightarrow \frac{dy}{dx} = \frac{1}{\sec x + \tan x} (\sec x \tan x + \sec^2 x)$$

$$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x} = \sec x$$

$$11. \quad y = x^3 \ln 2x \Rightarrow \frac{dy}{dx} = x^3 \left(\frac{2}{2x}\right) + 3x^2 \ln 2x = x^2(1 + 3 \ln 2x)$$

$$13. \quad y = \tan^{-1}(\ln x) \Rightarrow \frac{dy}{dx} = \frac{1}{1 + \ln^2 x} \left(\frac{1}{x}\right) = \frac{1}{x(1 + \ln^2 x)}$$

$$15. \quad y = x^2 \ln(x^2) \Rightarrow \frac{dy}{dx} = x^2 \left(\frac{2x}{x^2}\right) + 2x \ln(x^2) = 2x[1 + \ln(x^2)]$$

$$17. \quad y = \ln x - \frac{1}{2} \ln(1 + x^2) - \frac{\tan^{-1} x}{x}$$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{2} \left(\frac{2x}{1 + x^2}\right) - \frac{1}{x} \left(\frac{1}{1 + x^2}\right) - (\tan^{-1} x) \left(-\frac{1}{x^2}\right)$$

$$= \frac{1}{x(1 + x^2)} - \frac{1}{x(1 + x^2)} + \frac{\tan^{-1} x}{x} = \frac{\tan^{-1} x}{x}$$

$$19. \quad y = x[\sin(\ln x) + \cos(\ln x)]$$

$$\frac{dy}{dx} = [\sin(\ln x) + \cos(\ln x)] + x \left[\cos(\ln x) \left(\frac{1}{x}\right) - \sin(\ln x) \left(\frac{1}{x}\right) \right]$$

$$= 2\cos(\ln x)$$

$$21. \quad \int \frac{dx}{x} = \ln|x| + C$$

$$23. \quad \int \frac{dx}{2x} = \frac{1}{2} \ln|x| + C$$

$$25. \int_0^1 \frac{dx}{x+1} = \ln|x+1| \Big|_0^1 = \ln 2 - \ln 1 = \ln 2$$

$$27. \int_{-1}^0 \frac{dx}{2x+3} = \frac{1}{2} \ln|2x+3| \Big|_{-1}^0 = \frac{1}{2} [\ln 3 - \ln 1] = \frac{1}{2} \ln 3$$

$$29. \int_0^1 \frac{x dx}{4x^2+1} = \frac{1}{8} \ln(4x^2+1) \Big|_0^1 = \frac{1}{8} \ln 5$$

$$31. \int \tan 3x \, dx = \int \frac{\sin 3x}{\cos 3x} \, dx = -\frac{1}{3} \ln|\cos 3x| + C$$

$$33. \int \frac{x^2 \, dx}{4-x^3} = -\frac{1}{3} \ln|4-x^3| + C$$

$$35. \int \frac{dx}{x \ln x} = \int \frac{du}{u} = \ln|u| + C = \ln|\ln x| + C$$

$$\text{Let } u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$37. \int_1^2 \frac{(\ln x)^2}{x} \, dx = \int_0^{\ln 2} u^2 \, du = \frac{1}{3} u^3 \Big|_0^{\ln 2} = \frac{1}{3} \ln^3 2$$

$$\text{Let } u = \ln x \Rightarrow du = \frac{1}{x} dx; x=1 \Rightarrow u = \ln 1 = 0; x=2 \Rightarrow u = \ln 2$$

$$39. \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx = \int u^{-1} \, du = \ln|u| + C = \ln|\sec x + \tan x| + C$$

$$\text{Let } u = \sec x + \tan x \Rightarrow du = \sec x \tan x + \sec^2 x \, dx$$

$$41. \lim_{x \rightarrow \infty} \frac{\ln x}{x} = \lim_{x \rightarrow \infty} \left(\frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0$$

$$43. \lim_{t \rightarrow 0} \frac{\ln(1+2t) - 2t}{t^2} = \lim_{t \rightarrow 0} \left(\frac{\frac{2}{1+2t} - 2}{2t} \right) = \lim_{t \rightarrow 0} \left(\frac{-4}{(1+2t)^2} \right) = -2$$

$$45. A = \int_1^2 \left(\frac{2}{x} - 1 \right) dx = 2 \ln|x| - x \Big|_1^2 = 2 \ln 2 - 1$$

$$47. A = \int_0^1 \frac{2}{1+x^2} dx = 2 \tan^{-1} x \Big|_0^1 = \frac{\pi}{2}$$

$$\bar{x} = \frac{2}{\pi} \int_0^1 x \left(\frac{2}{1+x^2} \right) dx = \frac{2}{\pi} \ln |1+x^2| \Big|_0^1 = \frac{2 \ln 2}{\pi}$$

$$\bar{y} = 0 \text{ by symmetry. } \therefore (\bar{x}, \bar{y}) = \left(\frac{2 \ln 2}{\pi}, 0 \right).$$

$$49. x = \ln(\sec t + \tan t) - \sin t \Rightarrow \frac{dx}{dt} = \frac{\sec t \tan t + \sec^2 t}{\sec t + \tan t} - \cos t$$

$$\left(\frac{dx}{dt} \right)^2 = (\sec t - \cos t)^2 = \sec^2 t - 2 + \cos^2 t.$$

$$y = \cos t \Rightarrow \frac{dy}{dt} = -\sin t \Rightarrow \left(\frac{dy}{dt} \right)^2 = \sin^2 t$$

$$s = \int_0^{\frac{\pi}{3}} \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2} dt = \int_0^{\frac{\pi}{3}} \sqrt{\sec^2 t - 2 + \cos^2 t + \sin^2 t} dt$$

$$\int_0^{\frac{\pi}{3}} \sqrt{\sec^2 t - 1} dt = \int_0^{\frac{\pi}{3}} \tan t dt = \int_0^{\frac{\pi}{3}} \frac{\sin t}{\cos t} dt =$$

$$-\ln |\cos t| \Big|_0^{\frac{\pi}{3}} = -\left[\ln \frac{1}{2} - \ln 1 \right] = -\ln \frac{1}{2} = \ln 2$$

6.5 PROPERTIES OF THE NATURAL LOGARITHM

$$1. \ln 16 = \ln 2^4 = 4 \ln 2$$

$$3. \ln 2\sqrt{2} = \ln 2^{\frac{3}{2}} = \frac{3}{2} \ln 2$$

$$5. \ln \frac{4}{9} = \ln 4 - \ln 9 = 2 \ln 2 - 2 \ln 3$$

$$7. \ln \frac{9}{8} = \ln 9 - \ln 8 = 2 \ln 3 - 3 \ln 2$$

$$9. \ln 4.5 = \ln \frac{9}{2} = \ln 9 - \ln 2 = 2 \ln 3 - \ln 2$$

$$11. y = \ln \sqrt{x^2 + 5} = \frac{1}{2} \ln(x^2 + 5) \Rightarrow \frac{dy}{dx} = \frac{1}{2} \left(\frac{2x}{x^2 + 5} \right) = \frac{x}{x^2 + 5}$$

$$13. y = \ln \frac{1}{x\sqrt{x+1}} = -\ln x\sqrt{x+1} = -(\ln x + \frac{1}{2} \ln(x^2 + 5))$$

$$\frac{dy}{dx} = - \left[\frac{1}{x} + \frac{1}{2(x+1)} \right]$$

$$\begin{aligned} 15. \quad y = \ln(\sin x \sin 2x) &\Rightarrow \frac{dy}{dx} = \frac{1}{\sin x \sin 2x} \cdot \frac{d}{dx}(\sin x \sin 2x) \\ &= \frac{\sin x(2\cos x) + (\sin 2x)\cos x}{\sin x \sin 2x} \\ &= \frac{2\cos 2x + 2\cos^2 x}{\sin 2x} \end{aligned}$$

$$17. \quad y = \ln(3x\sqrt{x+2}) = \ln 3x + \frac{1}{2} \ln(x+2)$$

$$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2(x+2)}$$

$$\begin{aligned} 19. \quad y &= \frac{1}{3} \ln \left(\frac{x^3}{1+x^3} \right) \\ &= \frac{1}{3} [\ln x^3 - \ln(1+x^3)] \\ \frac{dy}{dx} &= \frac{1}{3} \left[\frac{3x^2}{x^3} - \frac{3x^2}{1+x^3} \right] \end{aligned}$$

$$21. \quad y = \ln \frac{(x^2+1)^5}{\sqrt{1-x}} = 5\ln(x^2+1) - \frac{1}{2}\ln(1-x)$$

$$\frac{dy}{dx} = \frac{10x}{x^2+1} + \frac{1}{2(1-x)}$$

$$23. \quad y^2 = x(x+1), \quad x > 0$$

$$2\ln y = \ln x + \ln(x+1)$$

$$\frac{2}{y} \frac{dy}{dx} = \frac{1}{x} + \frac{1}{x+1} \Rightarrow \frac{dy}{dx} = \frac{y}{2} \left(\frac{1}{x} + \frac{1}{x+1} \right)$$

$$25. \quad y = \sqrt{x+2} \sin x \cos x, \quad 0 < x < \frac{\pi}{2} \Rightarrow \ln y = \frac{1}{2} \ln(x+2) + \ln \sin x + \ln \cos x$$

$$\begin{aligned} \frac{1}{y} \frac{dy}{dx} &= \frac{1}{2(x+2)} + \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} \\ \frac{dy}{dx} &= y \left[\frac{1}{2(x+2)} + \cot x - \tan x \right] \end{aligned}$$

6.5 Properties of Natural Logarithms

$$27. \quad y = \sqrt[3]{\frac{x(x-2)}{x^2+1}}, \quad x > 2 \Rightarrow \ln y = \frac{1}{3}[\ln x + \ln(x-2) - \ln(x^2+1)]$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{3} \left[\frac{1}{x} + \frac{1}{x-2} - \frac{2x}{x^2+1} \right] \Rightarrow \frac{dy}{dx} = \frac{y}{3} \left[\frac{1}{x} + \frac{1}{x-2} - \frac{2x}{x^2+1} \right]$$

$$29. \quad y = \sqrt{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}, \quad x > 2$$

$$\ln y = \frac{1}{3}[\ln x + \ln(x+1) + \ln(x-2) - \ln(x^2+1) - \ln(2x+3)]$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{3} \left[\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{2}{2x+3} \right]$$

$$\frac{dy}{dx} = \frac{y}{3} \left[\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{2}{2x+3} \right]$$

$$31. \quad \sqrt{y} = \frac{x^5 \tan^{-1} x}{(3-2x)\sqrt[3]{x}} \Rightarrow$$

$$\frac{1}{2} \ln y = 5 \ln x + \ln \tan^{-1} x - \ln(3-2x) - \frac{1}{3} \ln x$$

$$\frac{1}{2y} \frac{dy}{dx} = \frac{5}{x} + \frac{1}{\tan^{-1} x} \cdot \frac{1}{1+x^2} + \frac{2}{3-2x} - \frac{1}{3x}$$

$$\frac{dy}{dx} = 2y \left[\frac{14}{3x} + \frac{2}{3-2x} + \frac{1}{(1+x^2)\tan^{-1} x} \right]$$

$$33. \quad \int_{-1}^1 \frac{dx}{x+3} = \ln|x+3| \Big|_{-1}^1 = \ln 4 - \ln 2 = \ln 2$$

$$35. \quad \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x} \, dx = \ln|\sin x| \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \ln \sqrt{2}$$

$$37. \quad \int_2^4 \frac{2x-5}{x} \, dx = \int_2^4 \left(2 - \frac{5}{x} \right) dx = 2x - 5 \ln|x| \Big|_2^4 = 4 - 5 \ln 2$$

$$39. \quad (a) \quad \int_0^{\frac{3}{5}} \frac{x \, dx}{1-x^2} = -\frac{1}{2} \int_1^{\frac{16}{25}} \frac{du}{u} = -\frac{1}{2} \ln|u| \Big|_1^{\frac{16}{25}} = \ln \left(\frac{16}{25} \right)^{-1/2} = \ln \frac{5}{4}$$

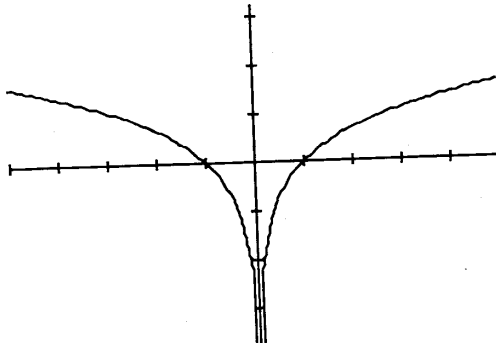
$$\text{Let } u = 1 - x^2 \Rightarrow du = -2x \, dx \Rightarrow x \, dx = -\frac{1}{2} du;$$

$$x = 0 \Rightarrow u = 1; \quad x = \frac{3}{5} \Rightarrow u = \frac{16}{25}$$

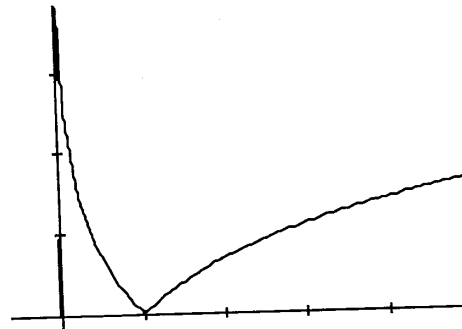
$$(b) \int_0^{\frac{3}{5}} \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x \Big|_0^{\frac{3}{5}} = \sin^{-1} \frac{3}{5}$$

$$(c) \int_0^{\frac{3}{5}} \frac{x dx}{\sqrt{1-x^2}} = -\sqrt{1-x^2} \Big|_0^{\frac{3}{5}} = \frac{1}{5}$$

41. (a) $y = \log|x|$



(b) $y = |\ln x|$



$$43. \lim_{x \rightarrow \infty} \int_x^{2x} \frac{1}{t} dt = \lim_{x \rightarrow \infty} [\ln t]_x^{2x} = \lim_{x \rightarrow \infty} [\ln 2x - \ln x] = \ln 2$$

45. The region is bounded by $y = \frac{4}{x}$ and $y = (x-3)^2$. These curves intersect where $\frac{4}{x} = (x-3)^2 \Leftrightarrow x^3 - 6x^2 + 9x - 4 = 0 \Leftrightarrow (x-1)^2(x-4) = 0$ or $x = 1, 4$. Then:

$$A = \int_1^4 \left[\frac{4}{x} - (x-3)^2 \right] dx = 4 \ln|x| - \frac{1}{3}(x-3)^2 \Big|_1^4 = 4 \ln 4 - 3$$

$$V = \pi \int_1^4 \left[\left(\frac{4}{x} \right)^2 - (x-3)^4 \right] dx = \pi \left[-\frac{16}{x} - \frac{1}{5}(x-3)^5 \right]_1^4 = \frac{27\pi}{5}$$

47. (a) If $f(x) = \ln(1+x)$, then $f'(x) = \frac{1}{1+x}$, $f''(x) = -\frac{1}{(1+x)^2}$ and

$$f'''(x) = \frac{2}{(1+x)^3}. \text{ Then } L(x) \approx f(0) + f'(0)x = \ln 1 + (1)x = x.$$

$$Q(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2}x^2 = \ln 1 + (1)x - \frac{1}{2}x^2 = x - \frac{x^2}{2}.$$

$$(b) \quad |e_1(x)| \leq \frac{1}{2}M_1x^2, \text{ where } M_1 = \max \{|f''(x)| : 0 \leq x \leq .1\} = 1$$

$$\therefore |e_1(x)| \leq \frac{1}{2}(1)(.1)^2 = 0.005$$

$$|e_2(x)| \leq \frac{1}{6}M_2|x|^3, \text{ where } M_2 = \max \{|f'''(x)| : 0 \leq x \leq .1\} = 2.$$

$$\therefore |e_2(x)| \leq \frac{1}{6}(2)(.1)^3 = 0.000\bar{3}$$

$$49. (a) \quad \ln 1.2 = \ln(1 + 0.2) \approx 0.2 \text{ by (16)}$$

$$\approx 0.2 - \frac{(.2)^2}{2} = 0.1800 \text{ by (17)}$$

$$\ln 1.2 \approx \frac{.1}{3} \left[1 + 4 \left(\frac{1}{1.1} \right) + \frac{1}{1.2} \right] \approx 0.18232 \text{ by Simpson's Rule.}$$

$$(b) \quad \ln .8 = \ln(1 - 0.2) \approx -0.2 \text{ by (16)}$$

$$\approx -0.2 - \frac{(-.2)^2}{2} = -0.2200 \text{ by (17)}$$

$$\ln .8 \approx -\frac{.1}{3} \left[\frac{1}{.8} + 4 \left(\frac{1}{.9} \right) + 1 \right] \approx -0.22315 \text{ by Simpson's Rule.}$$