

PROBLEMS

Evaluate the integrals in Problems 1–30.

1. $\int (x-1)^{2/3} dx$

2. $\int \sqrt{1-x} dx$

3. $\int \frac{1}{\sqrt{1-x}} dx$

4. $\int 2x\sqrt{x^2-1} dx$

5. $\int x\sqrt{2x^2-1} dx$

6. $\int (3x-1)^5 dx$

7. $\int (2-t)^{2/3} dt$

8. $\int x^2\sqrt{1+x^3} dx$

9. $\int (1+x^3)^2 dx$

10. $\int (1+x^3)^2 3x^2 dx$

11. $\int x(x^2+1)^{10} dx$

12. $\int \frac{dt}{2\sqrt{1+t}}$

13. $\int \frac{x^2}{\sqrt{1+x^3}} dx$

14. $\int \sqrt{2+5y} dy$

15. $\int \frac{dx}{(3x+2)^2}$

16. $\int 5r\sqrt{1-r^2} dr$

17. $\int \frac{3r dr}{\sqrt{1-r^2}}$

18. $\int \frac{y dy}{\sqrt{2y^2+1}}$

19. $\int x^4(7-x^5)^3 dx$

20. $\int \frac{x^3 dx}{\sqrt[4]{1+x^4}}$

21. $\int \frac{ds}{(s+1)^3}$

22. $\int \frac{s ds}{(s^2+1)^2}$

23. $\int \frac{1}{x^2+4x+4} dx$

24. $\int \frac{1}{y^2-2y+1} dy$

25. $\int \frac{x+1}{2\sqrt{x+1}} dx$

26. $\int x\sqrt{a^2-x^2} dx$

27. $\int (y^3+6y^2+12y+8)(y^2+4y+4) dy$

28. $\int \frac{(z+1) dz}{\sqrt[3]{z^2+2z+2}}$

29. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$

30. $\int (y^4+4y^2+1)^2(y^3+2y) dy$

Solve the differential equations in Problems 31–36 subject to the given initial conditions.

31. $\frac{dy}{dx} = x\sqrt{1+x^2}$, $y = 0$ when $x = 0$

32. $\frac{dy}{dx} = 3x^2\sqrt{1+x^3}$, $y = 1$ when $x = 0$

33. $\frac{dr}{dz} = 24z(3z^2-1)^3$, $r = -3$ when $z = 0$

34. $\frac{dy}{dx} = 4x(x^2-8)^{-1/3}$, $y = 0$ when $x = 0$

35. $2y \frac{dy}{dx} = 3x\sqrt{x^2+1}\sqrt{y^2+1}$, $y = 0$ when $x = 0$

36. $\frac{dy}{dx} = \frac{4\sqrt{(1+y^2)^3}}{y}$, $y = 0$ when $x = 0$

37. Which of the following methods could be used to evaluate

$$\int 3x^2(x^3-1)^5 dx?$$

- Expand $(x^3-1)^5$ and then multiply by $3x^2$ to get a polynomial to integrate term by term.
- Factor x^2 out to get an integral of the form $3x^2 \int u^n du$.
- Use the substitution $u = x^3-1$ to get an integral of the form $\int u^n du$.

4.3 SUBSTITUTION METHOD OF INTEGRATION

$$1. \int (x-1)^{243} dx = \frac{1}{244} (x-1)^{244} + C$$

$$3. \int \frac{1}{\sqrt{1-x}} dx = \int -u^{-\frac{1}{2}} du = -2u^{\frac{1}{2}} + C = -2(1-x)^{\frac{1}{2}} + C$$

$$\text{Let } u = 1 - x \Rightarrow du = -dx$$

$$5. \int x \sqrt{2x^2 - 1} dx = \int u^{\frac{1}{2}} \left(\frac{1}{4} du \right) = \frac{1}{4} \left(\frac{2}{3} u^{\frac{3}{2}} \right) + C = \frac{1}{6} (2x^2 - 1)^{\frac{3}{2}} + C$$

$$\text{Let } u = 2x^2 - 1 \Rightarrow du = 4x dx \Rightarrow x dx = \frac{1}{4} du$$

$$7. \int (2-t)^{\frac{2}{3}} dt = \int -u^{\frac{2}{3}} du = -\frac{3}{5} u^{\frac{5}{3}} + C = -\frac{3}{5} (2-t)^{\frac{5}{3}} + C$$

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$$9. \int (1+x^3)^2 dx = \int (1+2x^3+x^6) dx = x + \frac{1}{2}x^4 + \frac{1}{7}x^7 + C$$

$$11. \int x(x^2+1)^{10} dx = \int u^{10} \left(\frac{1}{2} du\right) = \frac{1}{2} \left(\frac{1}{11} u^{11}\right) + C = \frac{1}{22} (x^2+1)^{11} + C$$

$$\text{Let } u = x^2 + 1 \Rightarrow du = 2x dx \Rightarrow x dx = \frac{1}{2} du$$

$$13. \int \frac{x^2}{\sqrt{1+x^3}} dx = \int u^{-1/2} \left(\frac{1}{3} du\right) = \frac{1}{3} (2u^{1/2}) + C = \frac{2}{3} (1+x^3)^{1/2} + C$$

$$\text{Let } u = 1+x^3 \Rightarrow du = 3x^2 dx \Rightarrow x^2 dx = \frac{1}{3} du$$

$$15. \int \frac{dx}{(3x+2)^2} = \int u^{-2} \left(\frac{1}{3} du\right) = -\frac{1}{3} u^{-1} + C = -\frac{1}{3} (3x+2)^{-1} + C$$

$$\text{Let } u = 3x+2 \Rightarrow du = 3 dx \Rightarrow dx = \frac{1}{3} du$$

$$17. \int \frac{3r dr}{\sqrt{1-r^2}} = 3 \int u^{-1/2} \left(-\frac{1}{2} du\right) = -\frac{3}{2} (2u^{1/2}) + C = -3\sqrt{1-r^2} + C$$

$$\text{Let } u = 1-r^2 \Rightarrow du = -2r dr \Rightarrow r dr = -\frac{1}{2} du$$

$$19. \int x^4 (7-x^5)^3 dx = -\frac{1}{5} \int u^3 du = -\frac{1}{5} \left(\frac{1}{4} u^4\right) + C = -\frac{1}{20} (7-x^5)^4 + C$$

$$\text{Let } u = 7-x^5 \Rightarrow du = -5x^4 dx \Rightarrow x^4 dx = -\frac{1}{5} du$$

$$21. \int \frac{ds}{(s+1)^3} = \int (s+1)^{-3} ds = -\frac{1}{2} (s+1)^{-2} + C$$

$$23. \int \frac{1}{x^2+4x+4} dx = \int (x+2)^{-2} dx = -(x+2)^{-1} + C$$

$$25. \int \frac{x+1}{2\sqrt{x+1}} dx = \frac{1}{2} \int \frac{u}{\sqrt{u}} du = \frac{1}{2} \int u^{1/2} du = \frac{1}{3} u^{3/2} + C = \frac{1}{3} (x+1)^{3/2} + C$$

$$\text{Let } u = x+1 \Rightarrow du = dx$$

$$27. \int (y^3+6y^2+12y+8)(y^2+4y+4) dy = \int (y+2)^3 (y+2)^2 dy$$

$$= \int (y+2)^5 dy = \frac{1}{6} (y+2)^6 + C$$

$$29. \int \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx = \int \frac{1}{u^2} \cdot 2 du = \int 2u^{-2} du = -2u^{-1} + C = -2(1+\sqrt{x})^{-1} + C$$

$$\text{Let } u = 1+\sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}} dx \Rightarrow \frac{dx}{\sqrt{x}} = 2 du$$

$$31. \frac{dy}{dx} = x\sqrt{1+x^2}, y=0 \text{ when } x=0. y = \int x\sqrt{1+x^2} dx$$

$$= \frac{1}{3} (1+x^2)^{3/2} + C. 0 = \frac{1}{3} (1)^{3/2} + C \Rightarrow C = -\frac{1}{3}. \therefore y = \frac{1}{3} (1+x^2)^{3/2} - \frac{1}{3}.$$

$$33. \frac{dr}{dz} = 24z(3z^2 - 1)^3, r = -3 \text{ when } z = 0. r = \int 24z(3z^2 - 1)^3 dz$$

$$= (3z^2 - 1)^4 + C. -3 = (-1)^4 + C \Rightarrow C = -4. \therefore r = (3z^2 - 1)^4 - 4.$$

$$35. 2y \frac{dy}{dx} = 3x\sqrt{x^2 + 1}\sqrt{y^2 + 1}, y = 0 \text{ when } x = 0.$$

$$[(y^2 + 1)^{-\frac{1}{2}} y] dy = \frac{3}{2} x \sqrt{x^2 + 1} dx \Rightarrow (y^2 + 1)^{\frac{1}{2}} = \frac{1}{2}(x^2 + 1)^{\frac{3}{2}} + C.$$

$$(1)^{\frac{1}{2}} = \frac{1}{2}(1)^{\frac{3}{2}} + C \Rightarrow C = \frac{1}{2}. \therefore 2(y^2 + 1)^{\frac{1}{2}} = (x^2 + 1)^{\frac{3}{2}} + 1$$

37. Only (a) and (c). You are never allowed to factor a variable term out of the integral sign.

4.4 INTEGRALS OF TRIGONOMETRIC FUNCTIONS