PROBLEMS

Evaluate the integrals in Problems 1–30.

1. $\int (x-1)^{243}\,dx$

2. $\int \sqrt{1-x}\,dx$

3. $\int \frac{1}{\sqrt{1-x}}\,dx$

4. $\int 2x\sqrt{x^2-1}\,dx$

5. $\int x\sqrt{2x^2-1}\,dx$

6. $\int (3x-1)^5\,dx$

7. $\int (2-t)^{2/3}\,dt$

8. $\int x^2\sqrt{1+x^3}\,dx$

9. $\int (1+x^3)^2\,dx$

10. $\int (1+x^3)^2 3x^2\,dx$

11. $\int x(x^2+1)^{10}\,dx$

12. $\int \frac{dt}{2\sqrt{1+t}}$

13. $\int \frac{x^2}{\sqrt{1+x^3}}\,dx$

14. $\int \sqrt{2+5y}\,dy$

15. $\int \frac{dx}{(3x+2)^2}$

16. $\int 5r\sqrt{1-r^2}\,dr$

17. $\int \frac{3r\,dr}{\sqrt{1-r^2}}$

18. $\int \frac{y\,dy}{\sqrt{2y^2+1}}$

19. $\int x^4(7-x^5)^3\,dx$

20. $\int \frac{x^3\,dx}{\sqrt[4]{1+x^4}}$

21. $\int \frac{ds}{(s+1)^3}$

22. $\int \frac{s\,ds}{(s^2+1)^2}$

23. $\int \frac{1}{x^2+4x+4}\,dx$

24. $\int \frac{1}{y^2-2y+1}\,dy$

25. $\int \frac{x+1}{2\sqrt{x+1}}\,dx$

26. $\int x\sqrt{a^2-x^2}\,dx$

27. $\int (y^3+6y^2+12y+8)(y^2+4y+4)\,dy$

28. $\int \frac{(z+1)\,dz}{\sqrt[3]{z^2+2z+2}}$

29. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$

30. $\int (y^4+4y^2+1)^2(y^3+2y)\,dy$

Solve the differential equations in Problems 31–36 subject to the given initial conditions.

31. $\frac{dy}{dx} = x\sqrt{1+x^2}$, $y = 0$ when $x = 0$

32. $\frac{dy}{dx} = 3x^2\sqrt{1+x^3}$, $y = 1$ when $x = 0$

33. $\frac{dr}{dz} = 24z(3z^2-1)^3$, $r = -3$ when $z = 0$

34. $\frac{dy}{dx} = 4x(x^2-8)^{-1/3}$, $y = 0$ when $x = 0$

35. $2y\frac{dy}{dx} = 3x\sqrt{x^2+1}\sqrt{y^2+1}$, $y = 0$ when $x = 0$

36. $\frac{dy}{dx} = \frac{4\sqrt{(1+y^2)^3}}{y}$, $y = 0$ when $x = 0$

37. Which of the following methods could be used to evaluate

$$\int 3x^2(x^3-1)^5\,dx?$$

a) Expand $(x^3-1)^5$ and then multiply by $3x^2$ to get a polynomial to integrate term by term.
b) Factor x^2 out to get an integral of the form $3x^2\int u^n\,du$.
c) Use the substitution $u = x^3 - 1$ to get an integral of the form $\int u^n\,du$.

4.3 SUBSTITUTION METHOD OF INTEGRATION

1. $\int (x-1)^{243}\,dx = \frac{1}{244}(x-1)^{244} + C$

3. $\int \frac{1}{\sqrt{1-x}}\,dx = \int -u^{-\frac{1}{2}}\,du = -2u^{\frac{1}{2}} + C = -2\,(1-x)^{\frac{1}{2}} + C$

   Let $u = 1 - x \Rightarrow du = -dx$

5. $\int x\sqrt{2x^2 - 1}\,dx = \int u^{\frac{1}{2}}\left(\frac{1}{4}\,du\right) = \frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}}\right) + C = \frac{1}{6}(2x^2 - 1)^{\frac{3}{2}} + C$

   Let $u = 2x^2 - 1 \Rightarrow du = 4x\,dx \Rightarrow x\,dx = \frac{1}{4}\,du$

7. $\int (2-t)^{\frac{2}{3}}\,dt = \int -u^{\frac{2}{3}}\,du = -\frac{3}{5}u^{\frac{5}{3}} + C = -\frac{3}{5}\,(2-t)^{\frac{5}{3}} + C$

9. $\int (1+x^3)^2\,dx = \int (1+2x^3+x^6)\,dx = x + \frac{1}{2}x^4 + \frac{1}{7}x^7 + C$

11. $\int x(x^2+1)^{10}\,dx = \int u^{10}\left(\frac{1}{2}du\right) = \frac{1}{2}\left(\frac{1}{11}u^{11}\right) + C = \frac{1}{22}(x^2+1)^{11} + C$

Let $u = x^2+1 \Rightarrow du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}du$

13. $\int \frac{x^2}{\sqrt{1+x^3}}\,dx = \int u^{-1/2}\left(\frac{1}{3}du\right) = \frac{1}{3}(2u^{1/2}) + C = \frac{2}{3}(1+x^3)^{1/2} + C$

Let $u = 1+x^3 \Rightarrow du = 3x^2\,dx \Rightarrow x^2\,dx = \frac{1}{3}du$

15. $\int \frac{dx}{(3x+2)^2} = \int u^{-2}\left(\frac{1}{3}du\right) = -\frac{1}{3}u^{-1} + C = -\frac{1}{3}(3x+2)^{-1} + C$

Let $u = 3x+2 \Rightarrow du = 3\,dx \Rightarrow dx = \frac{1}{3}\,du$

17. $\int \frac{3r\,dr}{\sqrt{1-r^2}} = 3\int u^{-\frac{1}{2}}\left(-\frac{1}{2}du\right) = -\frac{3}{2}(2u^{\frac{1}{2}}) + C = -3\sqrt{1-r^2} + C$

Let $u = 1-r^2 \Rightarrow du = -2r\,dr \Rightarrow r\,dr = -\frac{1}{2}\,du$

19. $\int x^4(7-x^5)^3\,dx = -\frac{1}{5}\int u^3\,du = -\frac{1}{5}\left(\frac{1}{4}u^4\right) + C = -\frac{1}{20}(7-x^5)^4 + C$

Let $u = 7-x^5 \Rightarrow du = -5x^4\,dx \Rightarrow x^4\,dx = -\frac{1}{5}\,du$

21. $\int \frac{ds}{(s+1)^3} = \int (s+1)^{-3}\,ds = -\frac{1}{2}(s+1)^{-2} + C$

23. $\int \frac{1}{x^2+4x+4}\,dx = \int (x+2)^{-2}\,dx = -(x+2)^{-1} + C$

25. $\int \frac{x+1}{2\sqrt{x+1}}\,dx = \frac{1}{2}\int \frac{u}{\sqrt{u}}\,du = \frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{3}u^{\frac{3}{2}} + C = \frac{1}{3}(x+1)^{\frac{3}{2}} + C$

Let $u = x+1 \Rightarrow du = dx$

27. $\int (y^3+6y^2+12y+8)(y^2+4y+4)\,dy = \int (y+2)^3(y+2)^2\,dy$

$= \int (y+2)^5\,dy = \frac{1}{6}(y+2)^6 + C$

29. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx = \int \frac{1}{u^2}\cdot 2du = \int 2u^{-2}\,du = -2u^{-1} + C = -2(1+\sqrt{x})^{-1} + C$

Let $u = 1+\sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow \frac{dx}{\sqrt{x}} = 2du$

31. $\frac{dy}{dx} = x\sqrt{1+x^2}$, $y = 0$ when $x = 0$. $y = \int x\sqrt{1+x^2}\,dx$

$= \frac{1}{3}(1+x^2)^{3/2} + C.$ $0 = \frac{1}{3}(1)^{3/2} + C \Rightarrow C = -\frac{1}{3}.$ $\therefore y = \frac{1}{3}(1+x^2)^{3/2} - \frac{1}{3}.$

33. $\frac{dr}{dz} = 24z(3z^2 - 1)^3$, $r = -3$ when $z = 0$. $r = \int 24z(3z^2 - 1)^3\,dz$

$= (3z^2 - 1)^4 + C$. $-3 = (-1)^4 + C \Rightarrow C = -4$. $\therefore r = (3z^2 - 1)^4 - 4$.

35. $2y\frac{dy}{dx} = 3x\sqrt{x^2 + 1}\sqrt{y^2 + 1}$, $y = 0$ when $x = 0$.

$[(y^2 + 1)^{-\frac{1}{2}} y]\,dy = \frac{3}{2}x\sqrt{x^2 + 1}\,dx \Rightarrow (y^2 + 1)^{\frac{1}{2}} = \frac{1}{2}(x^2 + 1)^{\frac{3}{2}} + C$.

$(1)^{\frac{1}{2}} = \frac{1}{2}(1)^{\frac{3}{2}} + C \Rightarrow C = \frac{1}{2}$. $\therefore\ 2(y^2 + 1)^{\frac{1}{2}} = (x^2 + 1)^{\frac{3}{2}} + 1$

37. Only (a) and (c). You are never allowed to factor a variable term out of the integal sign.

4.4 INTEGRALS OF TRIGONOMETRIC FUNCTIONS