PROBLEMS

Evaluate the integrals in Problems 1–18.

1. a) $\int_0^3 \sqrt{y+1}\, dy$ b) $\int_{-1}^0 \sqrt{y+1}\, dy$

2. a) $\int_0^1 r\sqrt{1-r^2}\, dr$ b) $\int_{-1}^1 r\sqrt{1-r^2}\, dr$

3. a) $\int_0^{\pi/4} \tan x \sec^2 x\, dx$ b) $\int_{-\pi/4}^0 \tan x \sec^2 x\, dx$

4. a) $\int_0^1 x^3(1+x^4)^3\, dx$ b) $\int_{-1}^1 x^3(1+x^4)^3\, dx$

5. a) $\int_0^1 \frac{x^3}{\sqrt{x^4+9}}\, dx$ b) $\int_{-1}^0 \frac{x^3}{\sqrt{x^4+9}}\, dx$

6. a) $\int_{-1}^1 \frac{x}{(1+x^2)^2}\, dx$ b) $\int_0^1 \frac{x}{(1+x^2)^2}\, dx$

7. a) $\int_0^{\sqrt{7}} x(x^2+1)^{1/3}\, dx$ b) $\int_{-\sqrt{7}}^0 x(x^2+1)^{1/3}\, dx$

8. a) $\int_0^{\pi} 3\cos^2 x \sin x\, dx$ b) $\int_{2\pi}^{3\pi} 3\cos^2 x \sin x\, dx$

9. a) $\int_0^{\pi/6} (1-\cos 3x)\sin 3x\, dx$ b) $\int_{\pi/6}^{\pi/3} (1-\cos 3x)\sin 3x\, dx$

10. a) $\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\, dx$ b) $\int_{-\sqrt{3}}^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\, dx$

11. a) $\int_0^{2\pi} \frac{\cos x}{\sqrt{2+\sin x}}\, dx$ b) $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{2+\sin x}}\, dx$

12. a) $\int_{-\pi/2}^0 \frac{\sin x}{(3+\cos x)^2}\, dx$ b) $\int_0^{\pi/2} \frac{\sin x}{(3+\cos x)^2}\, dx$

13. a) $\int_{-\pi}^{\pi} x\cos(2x^2)\, dx$ b) $\int_{-\pi}^0 x\cos(2x^2)\, dx$

14. $\int_0^{\pi^2/4} \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$

15. $\int_0^1 \sqrt{t^5+2t}\,(5t^4+2)\, dt$

16. $\int_1^4 \frac{dy}{2\sqrt{y}\,(1+\sqrt{y})^2}$

17. $\int_0^{\pi/2} \cos^3 2x \sin 2x\, dx$

18. $\int_{-\pi/4}^{\pi/4} \tan^2 x \sec^2 x\, dx$

19. a) Graph the curve $y = x\sqrt{3-x^2}$.
    b) Find the area between the curve and the x-axis.

20. Evaluate

$$\int_0^{2\pi} |\cos x|\, dx.$$

21. Suppose $F(x)$ is an antiderivative of

$$f(x) = \frac{\sin x}{x}, \qquad x \neq 0.$$

Express

$$\int_1^3 \frac{\sin 2x}{x}\, dx$$

in terms of F.

22. Suppose that

$$\int_0^1 f(x)\, dx = 3.$$

Find

$$\int_{-1}^0 f(x)\, dx$$

if (a) f is odd, (b) f is even.

23. Suppose that the function $h(x)$ is even and continuous for all x.
    a) Show that the product $h(x)\sin x$ is odd.
    b) Show that, for any number a,

$$\int_{-a}^0 h(x)\sin x\, dx = -\int_0^a h(x)\sin x\, dx.$$

(*Hint:* Use the substitution $u = -x$.)

    c) Use the result in (b) to show that

$$\int_{-a}^a h(x)\sin x\, dx = 0.$$

    d) Show that

$$\int_{-\pi/4}^{\pi/4} \sec x \sin x\, dx = 0.$$

24. Show that

$$\int_{-a}^a h(x)\, dx = \begin{cases} 0 & \text{if } h \text{ is odd} \\ 2\int_0^a h(x)\, dx & \text{if } h \text{ is even.} \end{cases}$$

The shift property for definite integrals. A basic property of the definite integral is its invariance under translation, as expressed by the equation

$$\int_a^b f(x)\, dx = \int_{a-c}^{b-c} f(x+c)\, dx. \qquad (2)$$

This equation will hold whenever f is continuous and defined for the necessary values of x. For example,

$$\int_0^1 x^3\, dx = \int_{-2}^{-1} (x+2)^3\, dx = \int_2^3 (x-2)^3\, dx.$$

See Fig. 4.22.

1. (a) $\int_0^3 \sqrt{y+1}\,dy = \int_1^4 \sqrt{u}\,du = \frac{2}{3}u^{\frac{3}{2}}\Big]_1^4 = \frac{14}{3}$

(b) $\int_{-1}^0 \sqrt{y+1}\,dy = \int_0^1 \sqrt{u}\,du = \frac{2}{3}u^{\frac{3}{2}}\Big]_0^1 = \frac{2}{3}$

Let $u = y+1 \Rightarrow du = dy$. In (a) $y = 0 \Rightarrow u = 1$, $y = 3 \Rightarrow u = 4$.

In (b) $y = -1 \Rightarrow u = 0$, $y = 0 \Rightarrow u = 1$

3. (a) $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x\,dx = \int_0^1 u\,du = \frac{1}{2}u^2\Big]_0^1 = \frac{1}{2}$

(b) $\int_{-\frac{\pi}{4}}^0 \tan x \sec^2 x\,dx = \int_{-1}^0 u\,du = \frac{1}{2}u^2\Big]_{-1}^0 = -\frac{1}{2}$

Let $u = \tan x \Rightarrow du = \sec^2 x\,dx \Rightarrow$. Then in (a), $x = 0 \Rightarrow u = 0$,

$x = \frac{\pi}{4} \Rightarrow u = 1$, and in (b) $x = -\frac{\pi}{4} \Rightarrow u = -1$, $x = 0 \Rightarrow u = 0$.

5. (a) $\int_0^1 \frac{x^3}{\sqrt{x^4+9}}\,dx = \int_9^{10} \frac{1}{4}u^{-\frac{1}{2}}\,du = \frac{1}{2}u^{\frac{1}{2}}\Big]_9^{10} = \frac{\sqrt{10}-3}{2}$

(b) $\int_{-1}^0 \frac{x^3}{\sqrt{x^4+9}}\,dx = \int_{10}^9 \frac{1}{4}u^{-\frac{1}{2}}\,du = \frac{1}{2}u^{\frac{1}{2}}\Big]_{10}^9 = \frac{3-\sqrt{10}}{2}$

Let $u = x^4 + 9 \Rightarrow du = 4x^3\,dx \Rightarrow x^3\,dx = \frac{1}{4}\,du$. Then in (a), $x = 0 \Rightarrow u = 9$,

$x = 1 \Rightarrow u = 10$, , and in (b) $x = -1 \Rightarrow u = 10$, $x = 0 \Rightarrow u = 9$.

7. (a) $\int_0^{\sqrt{7}} x(x^2+1)^{\frac{1}{3}}\,dx = \int_1^8 \frac{1}{2}u^{\frac{1}{3}}\,du = \frac{3}{8}u^{\frac{4}{3}}\Big]_1^8 = \frac{45}{8}$

(b) $\int_{-\sqrt{7}}^0 x(x^2+1)^{\frac{1}{3}}\,dx = \int_8^1 \frac{1}{2}u^{\frac{1}{3}}\,du = \frac{3}{8}u^{\frac{4}{3}}\Big]_8^1 = -\frac{45}{8}$

Let $u = x^2 + 1 \Rightarrow du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}\,du$. Then in (a), $x = 0 \Rightarrow u = 1$, $x = \sqrt{7} \Rightarrow u = 8$, and in (b) $x = -\sqrt{7} \Rightarrow u = 8$, $x = 0 \Rightarrow u = 1$.

9. (a) $\int_0^{\frac{\pi}{6}} (1 - \cos 3x)\sin 3x\,dx = \int_0^1 \frac{1}{3}u\,du = \frac{1}{6}u^2\Big]_0^1 = \frac{1}{6}$

(b) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (1 - \cos 3x)\sin 3x\,dx = \int_1^2 \frac{1}{3}u\,du = \frac{1}{6}u^2\Big]_1^2 = \frac{1}{2}$

Let $u = 1 - \cos 3x \Rightarrow du = 3\sin 3x\,dx \Rightarrow \sin 3x\,dx = \frac{1}{3}u\,du$

Then in (a), $x = 0 \Rightarrow u = 0$, $x = \frac{\pi}{6} \Rightarrow u = 1$, and in (b) $x = \frac{\pi}{3} \Rightarrow u = 2$.

11. (a) $\int_0^{2\pi} \frac{\cos x\,dx}{\sqrt{2 + \sin x}} = 2\sqrt{2 + \sin x}\Big]_0^{2\pi} = 0$

(b) $\int_{-\pi}^{\pi} \frac{\cos x\,dx}{\sqrt{2 + \sin x}} = 2\sqrt{2 + \sin x}\Big]_{-\pi}^{\pi} = 0$

13. (a) $\int_{-\pi}^{\pi} x\cos(2x^2)\,dx = \frac{1}{4}\sin(2x^2)\Big]_{-\pi}^{\pi} = 0$

(b) $\int_{-\pi}^{0} x\cos(2x^2)\,dx = \frac{1}{4}\sin(2x^2)\Big]_{-\pi}^{0} = -\frac{1}{4}\sin(2\pi^2)$

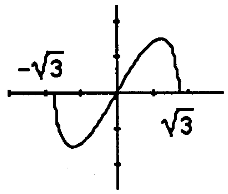
15. $\int_0^1 \sqrt{t^5 + 2t}\,(5t^4 + 2)\,dt = \frac{2}{3}(t^5 + 2t)^{\frac{3}{2}}\Big]_0^1 = 2\sqrt{3}$

17. $\int_0^{\frac{\pi}{2}} \cos^3 2x \sin 2x\,dx = \int_1^{-1} -\frac{1}{2}u^3\,du = -\frac{1}{8}u^4\Big]_1^{-1} = 0$

Let $u = \cos 2x \Rightarrow du = -2\sin 2x\,dx \Rightarrow \sin 2x\,dx = -\frac{1}{2}du$.

Then $x = 0 \Rightarrow u = 1$, $x = \frac{\pi}{2} \Rightarrow u = -1$.

19. (a)



(b) $2\int_0^{\sqrt{3}} x\sqrt{3-x^2}\,dx = -\frac{2}{3}(3-x^2)^{3/2}\Big]_0^{\sqrt{3}} = 2\sqrt{3}$

21. $\int_1^3 \frac{\sin 2x}{x}\,dx = 2\int_1^3 \frac{\sin 2x}{2x}dx = \int_2^6 \frac{\sin u}{u}\,du$, where we let

$u = 2x \Rightarrow \frac{1}{2}du = dx$ and $x = 1 \Rightarrow u = 2$, $x = 3 \Rightarrow u = 6$.

Therefore, $\int_1^3 \frac{\sin 2x}{x}\,dx = F(6) - F(2)$.

23. (a) Let $F(x) = h(x)\sin x$. Then $F(-x) = h(-x)\sin(-x) = h(x)(-\sin x) = -h(x)\sin x = -F(x)$. Therefore, the function i

(b) Let $u = -x$ so that $dx = -du$, and $x = 0 \Rightarrow u = 0$, $x = -a \Rightarrow u = a$

Then $\int_{-a}^{0} h(x)\sin x\,dx = -\int_0^a h(x)\sin x\,dx = \int_a^0 h(-u)\sin(-u)(-du)$

$= \int_a^0 h(u)\sin(u)\,du = -\int_0^a h(x)\sin x\,dx$.

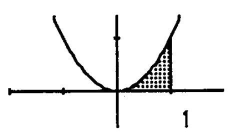
(c) $\int_{-a}^{a} h(x)\sin x\,dx = \int_{-a}^{0} h(x)\sin x\,dx + \int_0^a h(x)\sin x\,dx$

$= -\int_0^a h(x)\sin x\,dx + \int_0^a h(x)\sin x\,dx = 0$.

(d) $h(x) = \sec x$ is even, $a = \frac{\pi}{4}$, and $-a = -\frac{\pi}{4}$. By part (c)

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec x \sin x\,dx = 0$.

25. $\int_0^1 x^2\,dx = \frac{1}{3}x^3\Big]_0^1 = \frac{1}{3}$



$\int_{-1}^{0} (x+1)^2\,dx = \frac{1}{3}(x+1)^3\Big]_{-1}^{0} = \frac{1}{3}$