

PROBLEMS

Evaluate the integrals in Problems 1–18.

1. a) $\int_0^3 \sqrt{y+1} dy$ b) $\int_{-1}^0 \sqrt{y+1} dy$
2. a) $\int_0^1 r\sqrt{1-r^2} dr$ b) $\int_{-1}^1 r\sqrt{1-r^2} dr$
3. a) $\int_0^{\pi/4} \tan x \sec^2 x dx$ b) $\int_{-\pi/4}^0 \tan x \sec^2 x dx$
4. a) $\int_0^1 x^3(1+x^4)^3 dx$ b) $\int_{-1}^1 x^3(1+x^4)^3 dx$
5. a) $\int_0^1 \frac{x^3}{\sqrt{x^4+9}} dx$ b) $\int_{-1}^0 \frac{x^3}{\sqrt{x^4+9}} dx$
6. a) $\int_{-1}^1 \frac{x}{(1+x^2)^2} dx$ b) $\int_0^1 \frac{x}{(1+x^2)^2} dx$
7. a) $\int_0^{\sqrt{7}} x(x^2+1)^{1/3} dx$ b) $\int_{-\sqrt{7}}^0 x(x^2+1)^{1/3} dx$
8. a) $\int_0^{\pi} 3 \cos^2 x \sin x dx$ b) $\int_{2\pi}^{3\pi} 3 \cos^2 x \sin x dx$
9. a) $\int_0^{\pi/6} (1 - \cos 3x) \sin 3x dx$ b) $\int_{\pi/6}^{\pi/3} (1 - \cos 3x) \sin 3x dx$
10. a) $\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}} dx$ b) $\int_{-\sqrt{3}}^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}} dx$
11. a) $\int_0^{2\pi} \frac{\cos x}{\sqrt{2+\sin x}} dx$ b) $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{2+\sin x}} dx$
12. a) $\int_{-\pi/2}^0 \frac{\sin x}{(3+\cos x)^2} dx$ b) $\int_0^{\pi/2} \frac{\sin x}{(3+\cos x)^2} dx$
13. a) $\int_{-\pi}^{\pi} x \cos(2x^2) dx$ b) $\int_{-\pi}^0 x \cos(2x^2) dx$
14. $\int_0^{\pi^2/4} \frac{\cos \sqrt{x}}{\sqrt{x}} dx$
15. $\int_0^1 \sqrt{t^5+2t} (5t^4+2) dt$
16. $\int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2}$
17. $\int_0^{\pi/2} \cos^3 2x \sin 2x dx$
18. $\int_{-\pi/4}^{\pi/4} \tan^2 x \sec^2 x dx$
19. a) Graph the curve $y = x\sqrt{3-x^2}$.
 b) Find the area between the curve and the x -axis.

20. Evaluate

$$\int_0^{2\pi} |\cos x| dx.$$

21. Suppose $F(x)$ is an antiderivative of

$$f(x) = \frac{\sin x}{x}, \quad x \neq 0.$$

Express

$$\int_1^3 \frac{\sin 2x}{x} dx$$

in terms of F .

22. Suppose that

$$\int_0^1 f(x) dx = 3.$$

Find

$$\int_{-1}^0 f(x) dx$$

if (a) f is odd, (b) f is even.

23. Suppose that the function $h(x)$ is even and continuous for all x .

a) Show that the product $h(x) \sin x$ is odd.

b) Show that, for any number a ,

$$\int_{-a}^0 h(x) \sin x dx = -\int_0^a h(x) \sin x dx.$$

(Hint: Use the substitution $u = -x$.)

c) Use the result in (b) to show that

$$\int_{-a}^a h(x) \sin x dx = 0.$$

d) Show that

$$\int_{-\pi/4}^{\pi/4} \sec x \sin x dx = 0.$$

24. Show that

$$\int_{-a}^a h(x) dx = \begin{cases} 0 & \text{if } h \text{ is odd} \\ 2 \int_0^a h(x) dx & \text{if } h \text{ is even.} \end{cases}$$

The shift property for definite integrals. A basic property of the definite integral is its invariance under translation, as expressed by the equation

$$\int_a^b f(x) dx = \int_{a-c}^{b-c} f(x+c) dx. \quad (2)$$

This equation will hold whenever f is continuous and defined for the necessary values of x . For example,

$$\int_0^1 x^3 dx = \int_{-2}^{-1} (x+2)^3 dx = \int_2^3 (x-2)^3 dx.$$

See Fig. 4.22.

$$1. \quad (a) \int_0^3 \sqrt{y+1} \, dy = \int_1^4 \sqrt{u} \, du = \left. \frac{2}{3} u^{\frac{3}{2}} \right|_1^4 = \frac{14}{3}$$

$$(b) \int_{-1}^0 \sqrt{y+1} \, dy = \int_0^1 \sqrt{u} \, du = \left. \frac{2}{3} u^{\frac{3}{2}} \right|_0^1 = \frac{2}{3}$$

Let $u = y + 1 \Rightarrow du = dy$. In (a) $y = 0 \Rightarrow u = 1$, $y = 3 \Rightarrow u = 4$.

In (b) $y = -1 \Rightarrow u = 0$, $y = 0 \Rightarrow u = 1$

$$3. \quad (a) \int_0^{\frac{\pi}{4}} \tan x \sec^2 x \, dx = \int_0^1 u \, du = \left. \frac{1}{2} u^2 \right|_0^1 = \frac{1}{2}$$

$$(b) \int_{-\frac{\pi}{4}}^0 \tan x \sec^2 x \, dx = \int_{-1}^0 u \, du = \left. \frac{1}{2} u^2 \right|_{-1}^0 = -\frac{1}{2}$$

Let $u = \tan x \Rightarrow du = \sec^2 x \, dx \Rightarrow$. Then in (a), $x = 0 \Rightarrow u = 0$,

$x = \frac{\pi}{4} \Rightarrow u = 1$, and in (b) $x = -\frac{\pi}{4} \Rightarrow u = -1$, $x = 0 \Rightarrow u = 0$.

$$5. \quad (a) \int_0^1 \frac{x^3}{\sqrt{x^4+9}} \, dx = \int_9^{10} \frac{1}{4} u^{-\frac{1}{2}} \, du = \left. \frac{1}{2} u^{\frac{1}{2}} \right|_9^{10} = \frac{\sqrt{10}-3}{2}$$

$$(b) \int_{-1}^0 \frac{x^3}{\sqrt{x^4+9}} \, dx = \int_{10}^9 \frac{1}{4} u^{-\frac{1}{2}} \, du = \left. \frac{1}{2} u^{\frac{1}{2}} \right|_{10}^9 = \frac{3-\sqrt{10}}{2}$$

Let $u = x^4 + 9 \Rightarrow du = 4x^3 \, dx \Rightarrow x^3 \, dx = \frac{1}{4} du$. Then in (a), $x = 0 \Rightarrow u = 9$,

$x = 1 \Rightarrow u = 10$, , and in (b) $x = -1 \Rightarrow u = 10$, $x = 0 \Rightarrow u = 9$.

$$7. \quad (a) \int_0^{\sqrt{7}} x(x^2 + 1)^{\frac{1}{3}} dx = \int_1^8 \frac{1}{2} u^{\frac{1}{3}} du = \left. \frac{3}{8} u^{\frac{4}{3}} \right]_1^8 = \frac{45}{8}$$

$$(b) \int_{-\sqrt{7}}^0 x(x^2 + 1)^{\frac{1}{3}} dx = \int_8^1 \frac{1}{2} u^{\frac{1}{3}} du = \left. \frac{3}{8} u^{\frac{4}{3}} \right]_8^1 = -\frac{45}{8}$$

Let $u = x^2 + 1 \Rightarrow du = 2x dx \Rightarrow x dx = \frac{1}{2} du$. Then in (a), $x = 0 \Rightarrow u = 1$,

$x = \sqrt{7} \Rightarrow u = 8$, and in (b) $x = -\sqrt{7} \Rightarrow u = 8$, $x = 0 \Rightarrow u = 1$.

$$9. \quad (a) \int_0^{\frac{\pi}{6}} (1 - \cos 3x) \sin 3x dx = \int_0^1 \frac{1}{3} u du = \left. \frac{1}{6} u^2 \right]_0^1 = \frac{1}{6}$$

$$(b) \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (1 - \cos 3x) \sin 3x dx = \int_1^2 \frac{1}{3} u du = \left. \frac{1}{6} u^2 \right]_1^2 = \frac{1}{2}$$

Let $u = 1 - \cos 3x \Rightarrow du = 3 \sin 3x dx \Rightarrow \sin 3x dx = \frac{1}{3} u du$

Then in (a), $x = 0 \Rightarrow u = 0$, $x = \frac{\pi}{6} \Rightarrow u = 1$, and in (b) $x = \frac{\pi}{3} \Rightarrow u = 2$.

$$11. \quad (a) \int_0^{2\pi} \frac{\cos x dx}{\sqrt{2 + \sin x}} = \left. 2 \sqrt{2 + \sin x} \right]_0^{2\pi} = 0$$

$$(b) \int_{-\pi}^{\pi} \frac{\cos x dx}{\sqrt{2 + \sin x}} = \left. 2 \sqrt{2 + \sin x} \right]_{-\pi}^{\pi} = 0$$

$$13. \quad (a) \int_{-\pi}^{\pi} x \cos(2x^2) dx = \left. \frac{1}{4} \sin(2x^2) \right]_{-\pi}^{\pi} = 0$$

$$(b) \int_{-\pi}^0 x \cos(2x^2) dx = \left. \frac{1}{4} \sin(2x^2) \right]_{-\pi}^0 = -\frac{1}{4} \sin(2\pi^2)$$

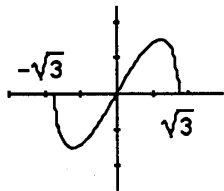
$$15. \quad \int_0^1 \sqrt{t^5 + 2t(5t^4 + 2)} dt = \left. \frac{2}{3} (t^5 + 2t)^{\frac{3}{2}} \right]_0^1 = 2\sqrt{3}$$

$$17. \quad \int_0^{\frac{\pi}{2}} \cos^3 2x \sin 2x dx = \int_1^{-1} -\frac{1}{2} u^3 du = \left. -\frac{1}{8} u^4 \right]_1^{-1} = 0$$

Let $u = \cos 2x \Rightarrow du = -2 \sin 2x dx \Rightarrow \sin 2x dx = -\frac{1}{2} du$.

Then $x = 0 \Rightarrow u = 1$, $x = \frac{\pi}{2} \Rightarrow u = -1$.

19. (a)



$$(b) 2 \int_0^{\sqrt{3}} x \sqrt{3-x^2} dx = -\frac{2}{3} (3-x^2)^{3/2} \Big|_0^{\sqrt{3}} = 2\sqrt{3}$$

21. $\int_1^3 \frac{\sin 2x}{x} dx = 2 \int_1^3 \frac{\sin 2x}{2x} dx = \int_2^6 \frac{\sin u}{u} du$, where we let

$$u = 2x \Rightarrow \frac{1}{2} du = dx \text{ and } x=1 \Rightarrow u=2, x=3 \Rightarrow u=6.$$

Therefore, $\int_1^3 \frac{\sin 2x}{x} dx = F(6) - F(2)$.

23. (a) Let $F(x) = h(x) \sin x$. Then $F(-x) = h(-x) \sin(-x) = h(x) (-\sin x) = -h(x) \sin x = -F(x)$. Therefore, the function is

(b) Let $u = -x$ so that $dx = -du$, and $x=0 \Rightarrow u=0$, $x=-a \Rightarrow u=a$

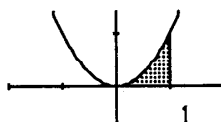
$$\begin{aligned} \text{Then } \int_{-a}^0 h(x) \sin x dx &= - \int_0^a h(x) \sin x dx = \int_a^0 h(-u) \sin(-u) (-du) \\ &= \int_a^0 h(u) \sin(u) du = - \int_0^a h(x) \sin x dx. \end{aligned}$$

$$\begin{aligned} (c) \int_{-a}^a h(x) \sin x dx &= \int_{-a}^0 h(x) \sin x dx + \int_0^a h(x) \sin x dx \\ &= - \int_0^a h(x) \sin x dx + \int_0^a h(x) \sin x dx = 0. \end{aligned}$$

(d) $h(x) = \sec x$ is even, $a = \frac{\pi}{4}$, and $-a = -\frac{\pi}{4}$. By part (c)

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec x \sin x dx = 0.$$

25. $\int_0^1 x^2 dx = \left. \frac{1}{3} x^3 \right|_0^1 = \frac{1}{3}$



$$\int_{-1}^0 (x+1)^2 dx = \left. \frac{1}{3} (x+1)^3 \right|_{-1}^0 = \frac{1}{3}$$

