

Derivatives of the Basic Trigonometric Functions

If u is a differentiable function of x , then:

$$1. \frac{d}{dx} \sin u = \cos u \frac{du}{dx},$$

$$2. \frac{d}{dx} \cos u = -\sin u \frac{du}{dx},$$

$$3. \frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx},$$

$$4. \frac{d}{dx} \sec u = \sec u \tan u \frac{du}{dx},$$

$$5. \frac{d}{dx} \csc u = -\csc u \cot u \frac{du}{dx},$$

$$6. \frac{d}{dx} \cot u = -\csc^2 u \frac{du}{dx}.$$

PROBLEMS

Problems 1–36, find dy/dx .

$$1. y = \sin(x + 1)$$

$$2. y = -\cos x$$

$$3. y = \sin(x/2)$$

$$4. y = \sin(-x)$$

$$5. y = \cos 5x$$

$$6. y = \cos(-x)$$

$$7. y = \cos(-2x)$$

$$8. y = \sin 7x$$

$$9. y = \sin(3x + 4)$$

$$10. y = \cos(2 - x)$$

$$11. y = x \sin x$$

$$12. y = \sin 5(x - 1)$$

$$13. y = x \sin x + \cos x$$

$$14. y = \frac{1}{\sin x}$$

$$15. y = \frac{1}{\cos x}$$

$$16. y = \frac{\sin x}{\cos x}$$

$$17. y = \sec(x - 1)$$

$$18. y = \cot(-x)$$

$$19. y = \sec(1 - x)$$

$$20. y = \frac{2}{\cos 3x}$$

$$21. y = \tan 2x$$

$$22. y = \cos(ax + b)$$

$$23. y = \sin^2 x$$

$$24. y = \sin^2 x + \cos^2 x$$

$$25. y = \cos^2 5x$$

$$26. y = \cot^2 x$$

$$27. y = \tan(5x - 1)$$

$$28. y = \sin x - x \cos x$$

$$29. y = 2 \sin x \cos x$$

$$30. y = \sec(x^2 + 1)$$

$$31. y = \sqrt{2 + \cos 2x}$$

$$32. y = \sin(1 - x^2)$$

$$33. y = \cos \sqrt{x}$$

$$34. y = \sec^2 x - \tan^2 x$$

$$35. y = \sqrt{\frac{1 + \cos 2x}{2}} \quad (\text{Hint: Use a half-angle formula first.})$$

$$36. y = \sin^2 x^2$$

Assume that each of the equations in Problems 37–41 defines y as a differentiable function of x . Find dy/dx by implicit differentiation.

$$37. x = \tan y$$

$$38. x = \sin y$$

$$39. y^2 = \sin^4 2x + \cos^4 2x$$

$$40. x + \sin y = xy$$

$$41. x + \tan(xy) = 0$$

42. Assume that the equation $2xy + \pi \sin y = 2\pi$ defines y as a differentiable function of x . Find dy/dx when $x = 1$ and $y = \pi/2$.

43. Find an equation for the tangent to the curve $x \sin 2y = y \cos 2x$ at the point $(\pi/4, \pi/2)$.

Find the limits in Problems 44–51.

$$44. \lim_{x \rightarrow 2} \sin\left(\frac{1}{x} - \frac{1}{2}\right)$$

$$45. \lim_{x \rightarrow \pi/4} \frac{\sin x}{\cos x}$$

$$46. \lim_{x \rightarrow -\pi} \cos^2 x$$

$$47. \lim_{x \rightarrow \pi} \sec(1 + \cos x)$$

$$48. \lim_{x \rightarrow 0} (\sec x + \tan x)$$

$$49. \lim_{x \rightarrow 0} x \csc x$$

$$50. \lim_{h \rightarrow 0} \frac{\sin(a + h) - \sin a}{h}$$

$$51. \lim_{h \rightarrow 0} \frac{\cos(a + h) + \cos a}{h}$$

52. Find an equation for the tangent to the curve $y = \sin mx$ at $x = 0$.

53. Graph $y = \tan x$ and its linearization $y = x$ together for $-\pi/4 \leq x \leq \pi/4$.

$$35. \quad \tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \cdot \frac{\frac{1}{\cos A \cos B}}{\frac{1}{\cos A \cos B}}$$

$$\frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \tan(A+(-B)) = \frac{\tan A + \tan(-B)}{1 - \tan A \tan(-B)} = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

2.7 DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

$$1. \quad y = \sin(x + 1) \Rightarrow y' = \cos(x + 1)$$

$$3. \quad y = \sin\left(\frac{x}{2}\right) \Rightarrow y' = \frac{1}{2}\cos\left(\frac{x}{2}\right)$$

$$5. \quad y = \cos 5x \Rightarrow y' = -5\sin 5x$$

$$7. \quad y = \cos(-2x) = \cos(2x) \Rightarrow y' = -\sin(2x)(2) = -2\sin(2x)$$

$$9. \quad y = \sin(3x + 4) \Rightarrow y' = 3\cos(3x + 4)$$

$$11. \quad y = x \sin x \Rightarrow y' = x \cos x + \sin x$$

$$13. \quad y = x \sin x + \cos x \Rightarrow y' = x \cos x + \sin x - \sin x = x \cos x$$

$$15. \quad y = \frac{1}{\cos x} = \sec x \Rightarrow y' = \sec x \tan x$$

$$17. \quad y = \sec(x - 1) \Rightarrow y' = \sec(x - 1) \tan(x - 1)$$

$$19. \quad y = \sec(1 - x) \Rightarrow y' = -\sec(1 - x) \tan(1 - x)$$

$$21. \quad y = \tan 2x \Rightarrow y' = 2\sec^2 2x$$

$$23. \quad y = \sin^2 x \Rightarrow y' = 2\sin x \cos x = \sin 2x$$

$$25. \quad y = \cos^2 5x \Rightarrow y' = -10\cos 5x \sin 5x = -5\sin 10x$$

$$27. \quad y = \tan(5x-1) \Rightarrow y' = 5\sec^2(5x-1)$$

$$29. \quad y = 2\sin x \cos x = \sin 2x \Rightarrow y' = 2\cos 2x$$

$$31. \quad y = \sqrt{2 + \cos 2x} \Rightarrow y' = \frac{1}{2}(2 + \cos 2x)^{-\frac{1}{2}}(-2\sin 2x) = \frac{-\sin 2x}{\sqrt{2 + \cos 2x}}$$

$$33. y = \cos\sqrt{x} \Rightarrow \frac{dy}{dx} = -\sin\sqrt{x} \left(\frac{1}{2}x^{-1/2} \right) = -\frac{\sin\sqrt{x}}{2\sqrt{x}}$$

$$35. y = \sqrt{\frac{1 + \cos 2x}{2}} = \sqrt{\cos^2 x} = |\cos x| \Rightarrow \frac{dy}{dx} = -\sin x \text{ if } \cos x > 0$$

and $\sin x$ if $\cos x < 0$.

$$37. x = \tan y \Rightarrow 1 = \sec^2 y \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y$$

$$39. y^2 = \sin^4 2x + \cos^4 2x \Rightarrow$$

$$2y \frac{dy}{dx} = 4(\sin^3 2x)(\cos 2x)(2) + 4(\cos^3 2x)(-\sin 2x)(2)$$

$$= 8\sin 2x \cos 2x (\sin^2 2x - \cos^2 2x) = 4\sin 4x (-\cos 4x) = -2\sin 8x$$

$$\frac{dy}{dx} = -\frac{\sin 8x}{y}$$

$$41. x + \tan(xy) = 0 \Rightarrow 1 + \sec^2(xy) \left[y + x \frac{dy}{dx} \right] = 0$$

$$1 + y \sec^2(xy) = -x \sec^2(xy) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-(1 + y \sec^2(xy))}{x \sec^2(xy)}$$

$$\text{or } \frac{dy}{dx} = -\frac{1}{x} [\cos^2(xy) + y]$$

$$43. x \sin 2y = y \cos 2x \Rightarrow 2x \cos 2y \frac{dy}{dx} + \sin 2y = -2y \sin 2x + \cos 2x \frac{dy}{dx}$$

Evaluating at $(\frac{\pi}{4}, \frac{\pi}{2})$:

$$2\left(\frac{\pi}{4}\right)(\cos \pi) \frac{dy}{dx} + \sin \pi = -2\left(\frac{\pi}{2}\right) \sin \frac{\pi}{2} + \left(\cos \frac{\pi}{2}\right) \frac{dy}{dx}$$

$$-\frac{\pi}{2} \frac{dy}{dx} = -\pi \Rightarrow \frac{dy}{dx} = 2$$

$$y - \frac{\pi}{2} = 2\left(x - \frac{\pi}{4}\right) \Rightarrow y = 2x \text{ is the equation of the tangent.}$$

$$45. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x}{\cos x} = \lim_{x \rightarrow \frac{\pi}{4}} \tan x = 1$$

$$47. \lim_{x \rightarrow \pi} \sec(1 + \cos x) = \sec(1 + \cos \pi) = \sec(1 - 1) = \sec 0 = 1$$

$$49. \lim_{x \rightarrow 0} x \csc x = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$